Calculus III Quiz 3 Fall 2000 9/25/2000

Each problem is worth 5 points, show all work for partial credit.

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1. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^3}$.

$\sum_{n=1}^{\infty} \frac{x^n}{n^3}$ <u>rat. Test</u> $\underline{\lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{(n+1)^3}}{\frac{x^n}{n^3}} \right|} = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{x^n} \right| = \lim_{n\to\infty} \left| \frac{x \cdot n^3}{(n+1)^3} \right| = |x| \lim_{n\to\infty} \left| \frac{n^3}{(n+1)^3} \right|$

$= |x| \lim_{n\to\infty} \left| \frac{n/n}{\frac{n}{n}+\frac{1}{n}} \right| = \underline{|x| \cdot 1}$ Conv. for $\underline{|x|<1}$

$\boxed{\text{radius of conv} = 1}$

Excellent

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2. Given that the radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$ is 1, give the interval of convergence.

$\underline{1}$: $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}} = \frac{(-1)^n}{\sqrt[3]{n}} \Rightarrow$ <u>converges</u> by <u>alt. series</u>

$= \frac{(-1)^n (1)^n}{\sqrt[3]{n}}$

$\underline{-1}$: $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}} = \frac{1}{n^{1/3}} \Rightarrow$ <u>diverges</u> by <u>p-series</u> $1/3 < 1$

$= \frac{(-1)^n(-1)^n}{\sqrt[3]{n}}$

so $\underline{(-1, 1]}$

Excellent