Calculus III Quiz 4 Fall 2000 10/4/2000

Each problem is worth 5 points, show all work for partial credit.

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1. For the curve given by the parametric equations $x = t^2+t$, $y = t^2-t$, find all values of t for which the tangent to the curve is vertical.

$x = t^2+t$
$y = t^2-t$

tang is vert ⇒ denom = 0 Yes.

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t-1}{2t+1}$

$2t+1=0 \Rightarrow 2t=-1$
$\boxed{t=-1/2}$

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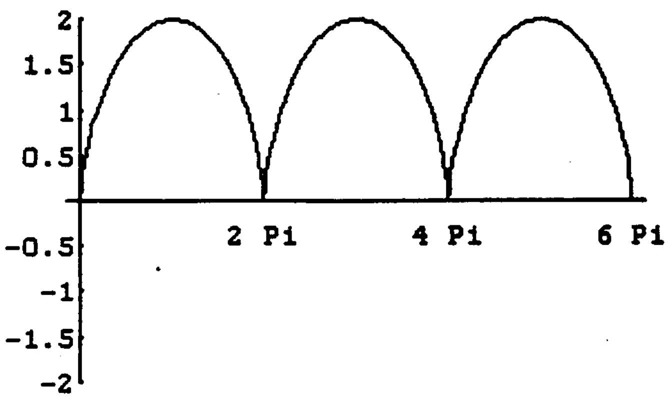
2. *Write* an integral for the area under *one arch* of the cycloid with parametric equations $x = t - \sin t$, $y = 1 - \cos t$. The plot below shows the curve for $0 \le t \le 6\pi$.



$\int_{t_0}^{t_f} y(t)\, x(t)'\, dt$

$\Rightarrow \boxed{\int_0^{2\pi} (1-\cos t)(1-\cos t)\, dt}$

simplified → or $\int_0^{2\pi} (1-2\cos t+\cos^2 t)\, dt$

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3. *Write* an integral for the length of *one arch* of the cycloid with parametric equations $x = t - \sin t$, $y = 1 - \cos t$, as shown in the graph above.

$x = t-\sin t$ $\frac{dx}{dt} = 1-\cos t$
$y = 1-\cos t$ $\frac{dy}{dt} = 0+\sin t = \sin t$

length of curve $= \int_\alpha^\beta \sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_\alpha^\beta \sqrt{(\sin t)^2+(1-\cos t)^2}\, dt$

$= \boxed{\int_0^{2\pi} \sqrt{(\sin t)^2+(1-\cos t)^2}\, dt}$

$= \int_{2\pi}^{4\pi} \sqrt{(\sin t)^2+(1-\cos t)^2}\, dt$

or $\int_{4\pi}^{6\pi} \sqrt{(\sin t)^2+(1-\cos t)^2}\, dt$