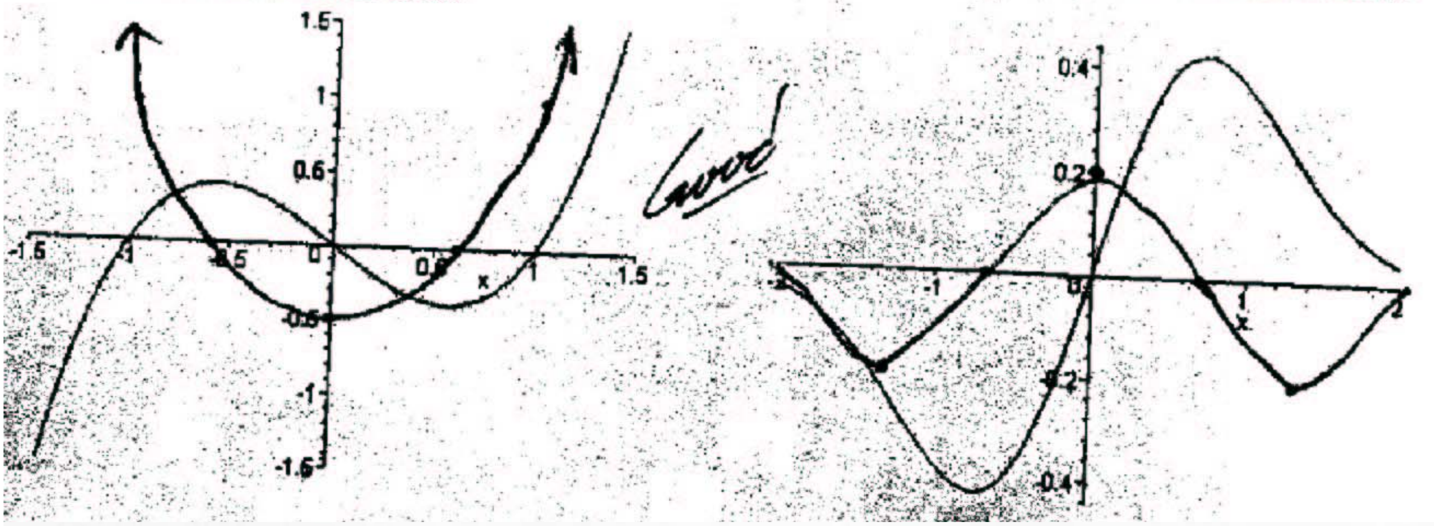
**Exam 1** **Calculus 1** **9/20/2002**

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Past returns do not assure future performance.

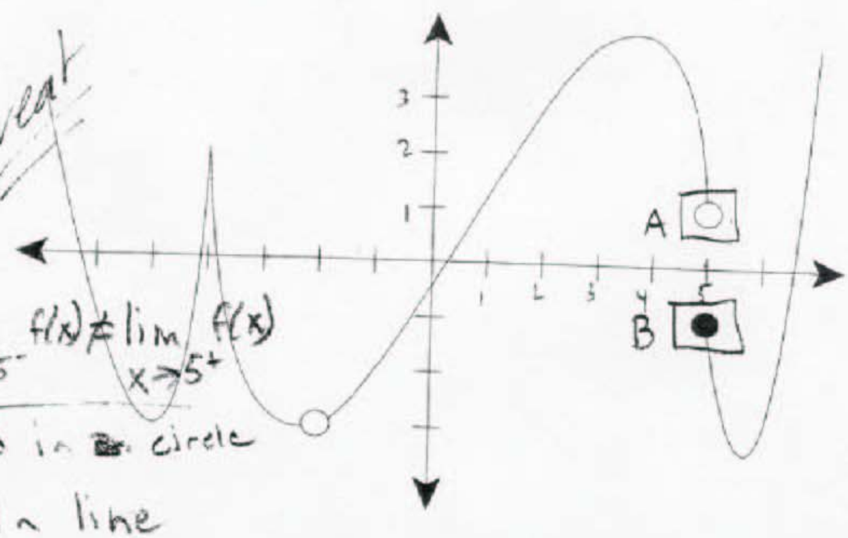
1. State the definition of the derivative $f'(x)$ of a function $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ Yes

**2. Two graphs are shown below. For each graph, sketch a graph of the derivative of the function shown on the same set of axes.**




Use the graph of f(x) shown at right for problems 3 and 4:




3. (a) What is $\lim_{x \to 5^-} f(x)$? = 1

Great

(b) What is $\lim_{x \to 5^+} f(x)$? -1

(c) What is $\lim_{x \to 5} f(x)$? = DNE $\lim_{x \to 5^-} f(x) \neq \lim_{x \to 5^+} f(x)$

(d) What is $f(5)$? = -1, use filled in circle

(e) What is f(-2)? DNE, hole in line

4. (a) For what values of x is f(x) discontinuous?

f(x) is discontinuous at x = -2, x = 5

(b) For what values of x is f(x) not differentiable?

Excellent

f(x) is not differentiable at x = -4, x = -2, x = 5

5. Let $f(x) = x^3 - 5x$. If you know that $f'(x) = 3x^2 - 5$, write an equation for the line tangent to f(x) when x=1.

(1, -4)

when x = 1

$f(x) = 1^3 - 5(1)$

$f(x) = 1 - 5$

$f(x) = -4$

$f'(1) = 3(1^2) - 5$

$f'(1) = 3 - 5$

$f'(1) = -2$ (which is slope)

$y - y_1 = m(x - x_1)$

$y + 4 = -2(x - 1)$

$y + 4 = -2x + 2 \rightarrow y = -2x - 2$

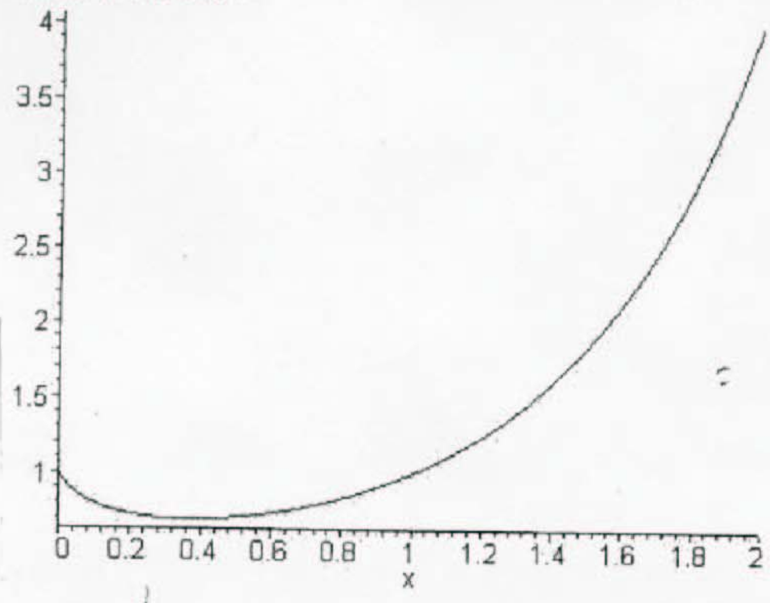
Very well done.

6. Compute $\lim_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right) \cdot \frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}}$

$$= \lim_{x\to\infty} \frac{(x^2+1)-(x^2-1)}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}} = 0$$

since that's 2 over a really big number.

7. A graph of $f(x) = x^x$ is shown below. Estimate the value of $f'(1.5)$ accurate to at least the nearest hundredth.




$x: 1.5001 \quad a = 1.5$

$$\frac{f(x) - f(a)}{x - a}$$

$$= \frac{1.5001^{1.5001} - 1.5^{1.5}}{1.5001 - 1.5}$$

$$= \frac{1.837375532 - 1.83711173}{0.0001}$$

$$= \frac{.000258224}{0.0001}$$

$$= 2.58$$

Excellent

8. Show that if $f(x) = \dfrac{1}{x}$, then $f'(x) = \dfrac{-1}{x^2}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x}{x}\left(\frac{1}{x+h}\right) - \frac{1}{x}\left(\frac{x+h}{x+h}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x - x - h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{x^2 + xh}\left(\frac{1}{h}\right)$$

$$= \lim_{h \to 0} \frac{-1}{x^2 + xh} = \frac{-1}{x^2}$$

Very well done

9. The land that Jon and Darcy's house is built on was appraised at $15,550 in 2001 and $32,480 in 2002. Darcy says "So the value went up by $16930 in just a year. If it keeps going like that, it'll be $49410 next year." Jon answers "No, it's even better than that, since it more than doubled in just a year, if it keeps going like that it'll be $67842 next year."

Explain clearly to Jon and Darcy what they should expect for their 2003 appraisal. [Yes, this is a vague question – but you've got the tools to give really good answers, so see what you can do.]

At this current trend, if it continues, we can see that the house had an increase of 1,089 it's value. To find the cost for the next year then, we take 32,480 and multiply it by 2.089 to get a total of $67,85.42 for the house. This would only be if the trend continues at that current rate and seeks no limit.

Very important last sentence!

10. Samantha is a calculus student at Enormous State University, and she's having some trouble with limits. Samantha says "So, like, I totally bombed this quiz we had about limits. We were supposed to say what the limit of things like $\frac{n-1}{n}$ was, and so I said I didn't really know, but for sure it had to be less than 1, because stuff like ½ and 2/3 and all that are less than 1. So whatever the limit is, it's gotta be less than 1 too, right? But the professor didn't really like it, I guess, and he wrote this long note I totally couldn't even read, and I got no points, so I really better figure this out for the exam, huh?"

Help Samantha by explaining, in terms she can understand, either how to convince her professor she's right, or how it is that terms less than 1 can have a limit equal to 1.

As $f(x) = \frac{n-1}{n}$ approaches infinity, That is $\lim_{n\to\infty} \frac{n-1}{n}$, The value will become progressively closer to 1. The larger n becomes, the closer to 1 this function will go. So, the $\lim_{n\to\infty} \frac{n-1}{n} = 1$.

For example:

$$\left.\begin{aligned} \frac{1000-1}{1000} &= 0.999 \\ \frac{10{,}000-1}{10{,}000} &= 0.9999 \\ \frac{1{,}000{,}000-1}{1{,}000{,}000} &= 0.99999 \end{aligned}\right\} \text{These values become closer to 1 each time } n \text{ gets closer to infinity.}$$

Excellent