Exam 3 Calculus 1 11/8/2002

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Keep arms within car at all times.

1. Find two numbers whose sum is 50 and whose product is a maximum.

Let one number be x so the other one is (50-x)

$S(x) = x(50-x)$

$S(x) = 50x - x^2$

$S'(x) = 50 - 2x$

$0 = 50 - 2x$

$\frac{2x}{2} = \frac{50}{2}$

$x = 25$ so $(50-x) = 25$

the two numbers are 25 + 25

Yes!

2. Jon gorges himself on leftover Halloween candy, and his body responds by releasing insulin into his bloodstream to reduce his blood sugar level back to normal. If the extra sugar in Jon's bloodstream after t hours is given by the function $f(t) = 17t\, e^{-15t}$, when is his sugar rush at its peak, that is, at exactly what t value does the maximum value of $f(t) = 17t\, e^{-15t}$ occur?

$f(t) = 17te^{-15t}$

fs'+f's

$f'(t) = (17t)(-15e^{-15t}) + 17e^{-15t}$

$f'(t) = -255te^{-15t} + 17e^{-15t}$

$f'(t) = 17e^{-15t}(-15t+1)$

$0 = 17e^{-15t}(-15t+1)$

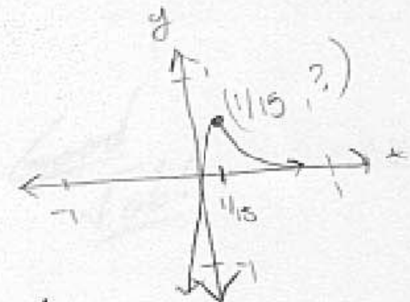
$0 = 17e^{-15t}$ or $-15t+1=0$

solver

$t=$ or $1 = 15t$

$t = 1/15$

Good



3. If $f'(x) = 6x^2 - 6x - 36$ [notice that prime!], where does f(x) have relative (local) extrema, and are they mins or maxes?

$f'(x) = 6x^2 - 6x - 36$

$0 = 6x^2 - 6x - 36$

$0 = 6(x^2 - x - 6)$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$\boxed{x = -2 \text{ and } x = 3}$

(+) (−) (+)

−2 3 x

$f'(4) = 6(4)^2 - 6(4) - 36 = 96 - 24 - 36 = 36$

$f'(0) = 6(0)^2 - 6(0) - 36 = -36$

$f'(-3) = 6(-3)^2 - 6(-3) - 36 = 54 + 18 - 36 = 36$

W

$\boxed{\begin{array}{l}\text{loc. max: } x = -2 \\ \text{loc. min: } x = 3\end{array}}$ ⟸ answer

Great!

$f''(x) = 12x - 6$

$0 = 12x - 6$

$0 = 6(2x - 1)$

$0 = 2x - 1$

$1 = 2x$

$x = \frac{1}{2}$

CD$(-\infty, \frac{1}{2})$ CU$(\frac{1}{2}, \infty)$

(−) (+)

$\frac{1}{2}$ x

$f''(1) = 12(1) - 6 = 6$

$f''(0) = 12(0) - 6 = -6$

4. Evaluate $\lim\limits_{x\to 0}\dfrac{e^x - 1 - x}{x^2}$.

$$\lim_{x\to 0}\frac{e^x - 1 - x}{x^2} \overset{L'H}{=} \lim_{x\to 0}\frac{\frac{d}{dx}e^x - \frac{d}{dx}1 - \frac{d}{dx}x}{\frac{d}{dx}x^2}$$

$$= \lim_{x\to 0}\frac{e^x - 1}{2x} \overset{L'H}{=} \lim_{x\to 0}\frac{\frac{d}{dx}e^x - \frac{d}{dx}1}{\frac{d}{dx}2x}$$

W

$$= \lim_{x\to 0}\frac{e^x}{2} = \frac{e^0}{2} = \boxed{\frac{1}{2}}$$

Good Job!

5. Use Newton's Method with an initial approximation $x_1 = 2$ to find x_2, the second approximation to $\sqrt[3]{5}$.

$x^3 = 5 \qquad f(x) = x^3 - 5 \qquad f'(x) = 3x^2$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = (2) - \frac{(2)^3 - 5}{3(2)^2} = 2 - \frac{3}{12} = \frac{7}{4}$$

Nice

6. Describe the behavior of $f(x) = \dfrac{2+x}{\sqrt{x}}$ near $x=0$.

$$f(0) = \frac{2+0}{\sqrt{0}}$$

$f(0)$ does not exist

$\lim_{x \to 0^-} f(x)$ does not exist

$\lim_{x \to 0^+} f(x) = \infty$

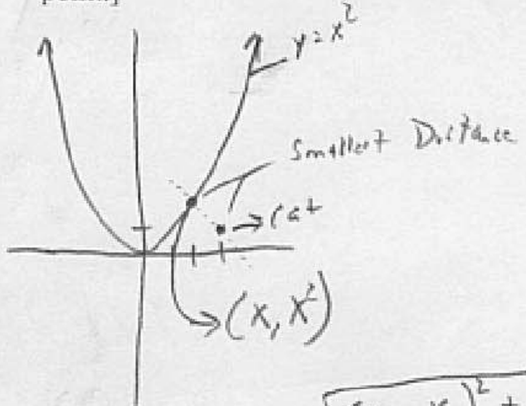
Well done

7. James B. Appleberry, President of the American Association of State Colleges and Universities, was quoted in *The Chronicle of Higher Education* on October 5, 1994 as saying "The good news is that the rate of increase has continued to lessen. The bad news is that any increase in tuition limits access." If $C(t)$ is a function giving the price of a college education in the year t, explain (in at least two complete sentences) what the quotation implies about the signs of $C'(1994)$ and $C''(1994)$.

The quotation is saying that the derivative of the function has a positive sign because the tuition is increasing. The 2nd derivative, however, has a negative sign because the function is increasing less and less.

Exactly!

8. Jon is dragging a toy mouse on a string past his cat Nemo. Nemo is a very smart cat, so he's going to wait to pounce until the mouse is at its closest. If Nemo is sitting at the point (3,1) and Jon drags the mouse along the curve $y = x^2$, what equation could Nemo solve in order to get the coordinates of that closest point on the mouse's path? [**You don't need to actually solve the equation**, just find an equation whose solution would give the x coordinate of the desired closest point.]



x_1, y_1
$(3, 1)$

$x_2 \quad y_2$
(x, x^2)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$d^2=(x_2-x_1)^2+(y_2-y_1)^2$$
$$d^2=(x-3)^2+(x^2-1)^2$$

eqn. to optimize distance $f(x)=(x-3)^2+(x^2-1)^2$ $\quad 4x(x^2-1)$

$$f'(x)=2(x-3)(1)+2(x^2-1)(2x)$$
$$=2x-6+4x^3-4x = \boxed{4x^3-2x-6=0}$$

Excellent

9. Jon is going to spend his summers designing mathematically perfect roller coasters, which he's pretty sure will be a lot more fun than regular roller coasters. He's trying to find a function to use for the main dip on his first creation, and he's decided he wants to use one shaped like a function of the form $f(x) = x^3 + bx^2$ for some constant b. If he wants the low point to come when x=5, what value should he use for b?

Low point... probably something like a local min...

I $f'(x) = 3x^2 + 2bx$

II $0 = 3x^2 + 2bx$

III plug in value for x and solve for b

$0 = 3(5)^2 + 2(b)(5)$

$-75 = 10b$

$b = \frac{-75}{10} = -7.5$

The value for b should be $\boxed{-7.5}$

Excellent

10. At what point(s) is the function $f(x) = \dfrac{x}{x^2+9}$ **decreasing** most rapidly?

[Head start: $f'(x) = \dfrac{9-x^2}{(x^2+9)^2}$ and $f''(x) = \dfrac{2x(x^2-27)}{(x^2+9)^3}$.]

f(x) is decreasing most rapidly when you maximize f'(x) → f''(x) = to zero and solving.

$$0 = \frac{2x(x^2-27)}{(x^2+9)^3}$$

$\cancel{x=0}$, $x = \sqrt{27}$, $x = -\sqrt{27}$

not decreasing here

decreasing most rapidly at $\left(\sqrt{27}, \frac{\sqrt{27}}{36}\right)$ and $\left(-\sqrt{27}, \frac{-\sqrt{27}}{36}\right)$

Well done!