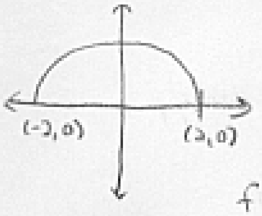
Quiz 4 Calc 3 11/26/2002

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x,y) = y^3\,\mathbf{i} + 3xy^2\,\mathbf{j}$ where C is the top half of a circle centered at the origin beginning at (2,0) and ending at (-2,0).

$F(x,y) = \langle y^3, 3xy^2 \rangle$ $\qquad f_x = y^3 \qquad f_y = 3xy^2$

$xy^3, \ \frac{3xy^3}{3}$ $\qquad f_{xy} = 3y^2 \qquad f_{yx} = 3y^2$



Since $f_{xy} = f_{yx}$ there is a potential function, use Fundamental Theorem.

$f(x,y) = xy^3$

$\int_C \vec{F} \cdot d\vec{r} = \left[xy^3 \right]_{(2,0)}^{(-2,0)}$

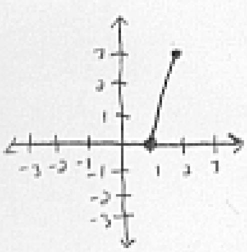
Great Job!

$[(-2)(0)^3 - (2)(0)^3] \rightarrow 0 - 0 = \boxed{0}$

2. Let $\mathbf{F}(x,y) = -y\mathbf{i} + x\mathbf{j}$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for C the line segment beginning at (1,0) and ending at (2,3).

$F(x,y) = \langle -y, x \rangle$ $\qquad f_x = -y \qquad f_y = x$

$f_{xy} = -1 \qquad f_{yx} = 1$



$f_{xy} \neq f_{yx}$ so can't use fundamental theorem.

$x = 1 + z$

$y = 0 + 3z$ $\qquad 0 \le z \le 1$

$r(z) = \langle 1 + z, 3z \rangle$

Nicely done!

$F(r(z)) = \langle -3z, 1+z \rangle$

$r'(z) = \langle 1, 3 \rangle$

$\int_C \vec{F} \cdot \vec{r}' \, dz = \int_0^1 \langle -3z, 1+z \rangle \cdot \langle 1, 3 \rangle \, dz$

$= \int_0^1 -3z + (3 + 3z) \, dz$

$= \int_0^1 3 \, dz$

$= [3z]_0^1$

$= 3 - 0 = \boxed{3}$