Problem Set 9 Real Analysis 1 Due 12/6/2002

Each problem is worth 5 points. Adequate demonstration is required for full credit.

1. Give an example of a function $f:\mathbb{R}\to\mathbb{R}$ for which an upper sum on [0,1] with four subdivisions gives three times the value of $\int_0^1 f\,dx$.

2. Give an example of a function $f:\mathbb{R}\to\mathbb{R}$ for which an lower sum on [0,1] with four subdivisions is zero even though the value of $\int_0^1 f\,dx$ is not zero.

3. Prove or give a counterexample: If functions f and g are not Riemann integrable, then f+g is not Riemann integrable.

4. Prove or give a counterexample: If $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function, then f is Riemann integrable on any interval [a,b].

5. Prove or give a counterexample: If $f:\mathbb{R}\to\mathbb{R}$ is a Riemann integrable function on any interval [a,b], then f is differentiable.

6. Prove that a constant function f(x) = c, for $c \in \mathbb{R}$, is Riemann integrable on any interval [a,b].