Exam 3 Algebra & Trig 11/11/2003

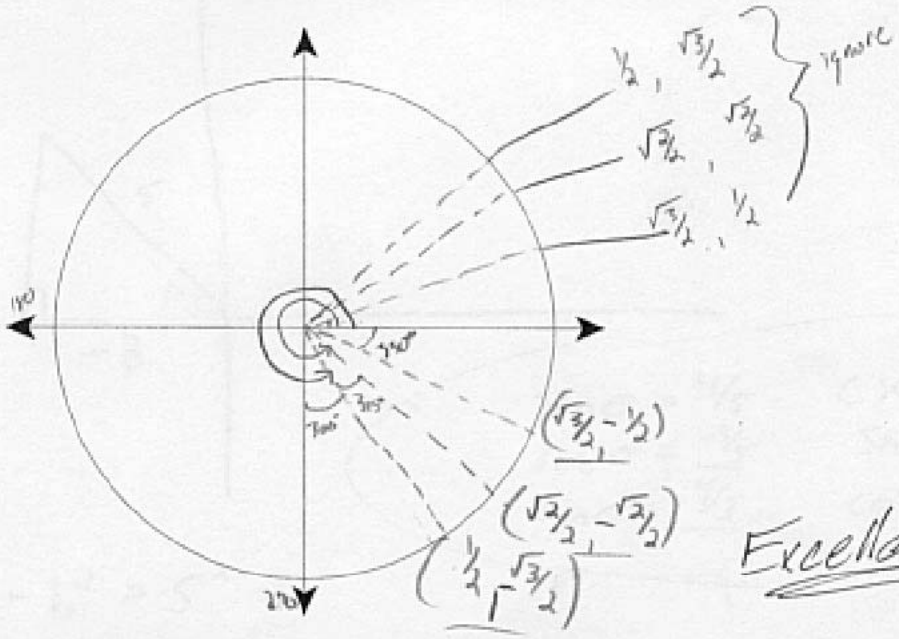
Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

1. Evaluate $\ln \frac{1}{e^7}$ exactly.

$\ln \frac{1}{e^7} = \ln e^{-7} =$

$\ln e^{-7} = -7$ Good!

2. On the unit circle shown below, mark the points corresponding to angles of 300°, 315°, and 330° (measured counterclockwise from the positive x axis) and label these points with their exact x and y coordinates.



Excellent!

3. Solve the equation $\ln x = 3$ exactly.

$\ln x = 3$

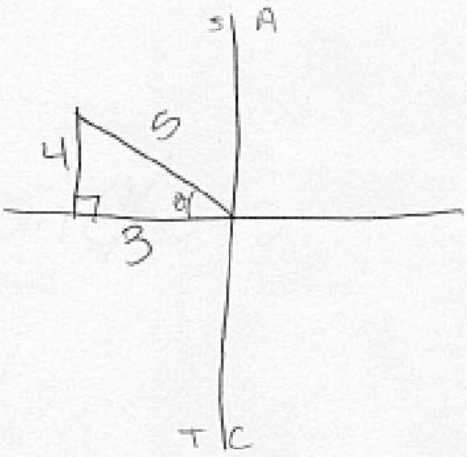
$e^{\ln x} = e^3$

$x = e^3$ Great

4. If θ is a second-quadrant angle for which sin θ = 4/5, give exact values for the other five trig functions of θ.

SOH CAH TOA

O = O
S = H
A = ±3



$\sin\theta = 4/5$

$\cos\theta = -3/5$

$\tan\theta = -4/3$

$\csc\theta = 5/4$

$\sec\theta = -5/3$

$\cot\theta = -3/4$

$a^2 + 4^2 = 5^2$

$16 + a^2 = 25 - 16$

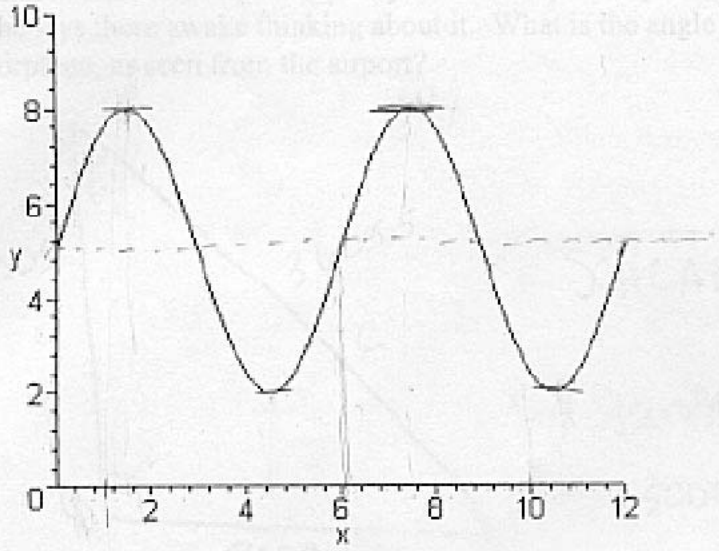
$\sqrt{a^2} = \sqrt{9}$

$a = \pm 3$

for this, since it is going to the left, the 3 will be negative

Exactly

5. Find a possible formula for the graph shown:



$y = A\sin(Bx + C) + D$

Average value of 5 ← D

Amplitude of 3 ← A

Started

period $= \frac{2\pi}{B}$

$\frac{2\pi}{6} \quad \frac{\pi}{3}$

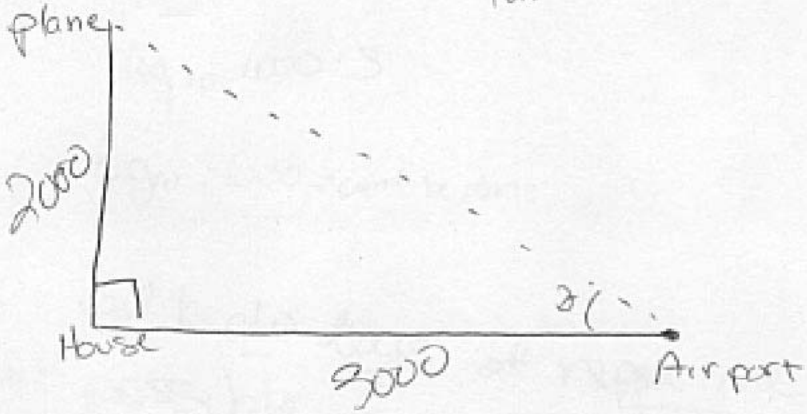
$y = 3\sin + 5$

$y = 3\sin\left(\frac{\pi}{3}x\right) + 5$

Excellent

6. In the middle of the night an airplane comes in to land at the Cedar Rapids airport. The plane is 2000 feet above Jon's house, which is 3000 feet from the airport (and the ground between Jon's house and the airport is pretty much level). The plane is really loud, so it wakes Jon up, and he lays there awake thinking about it. What is the angle between the horizontal ground and the airplane, as seen from the airport?

SOH
CAH
TOA



$3^2 + 2^2 = c^2$

$9 + 4 = 13^2$

$\tan\theta = \frac{2000}{3000}$

$\tan\theta = 2/3$

$\theta = \tan^{-1}(2/3)$ Yes!

7. Biff is having some trouble with logs. He says "So I screwed up this thing on my math test, and the tutor my parents are paying for couldn't really explain it to me. I did something and got a negative number for it, and then I was supposed to do the log of that, and my calculator I guess can't do those. And so I asked the tutor, and she said you just can't do logs of negatives. But that's crazy, right, because it's gotta be something. So what's up with logs of negatives?"

Explain clearly to Biff either how to do the log of a negative number, or why it's not possible to do so.

It's not possible to do the log of a negative number because for example if you had this $\log_{3}{-9}$ or $\log_{2}{-4}$. First you ask yourself what is the exponent you put on -3 or -2 to get -9 or -4. Well each of the answers would be 2, but wait when you multiply a positive by a positive there is no way of getting a negative log! It doesn't work. But don't get confused when you see something like this $\log_{2}(\frac{1}{25}) = -2$ you can get a negative answer for your exponent though.

Excellent

8. Kim manages to convince a really stupid bank that it'll be okay to give her a really nice interest rate of 24%, compounded monthly, as long as she only deposits $100 in the account. How much money will Kim have in the account if she leaves it there for 30 years?

$P(1+\frac{r}{n})^{nt}$

$100 = P$

$.24 = r$

$12 = n$

$t = 30$

$12\overline{)\,.24}$ → $.02$, $\frac{12}{24}$

$100(1+\frac{.24}{12})^{12(30)}$

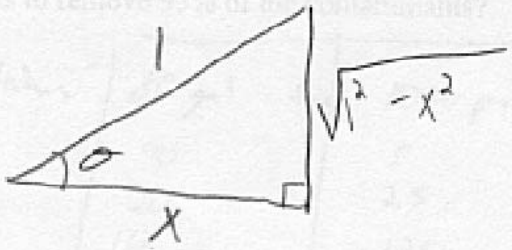
$12 \times 30 = 360$

$100(1+.02)^{360}$

$100\ (1.02)^{360}$ Exactly.

SOH CAH TOA

9. Simplify $\tan(\cos^{-1}\frac{x}{1})$.



$x^2 + b^2 = 1^2$

$b^2 = 1^2 - x^2$

$\sqrt{b^2} = \sqrt{1^2-x^2}$

$b = \sqrt{1^2-x^2}$

$\tan\theta = \frac{\sqrt{1^2-x^2}}{x}$

Well done

10. Suppose that a water filtration system will remove 80% of the contaminants in a gallon of water during each 10 hour period. How much longer will it take the system to remove 99% of the contaminants than it takes to remove 95% of the contaminants?

starting amount, whatever it is

removing 80% means leaving 20%

$$C(t) = C_0 (.2)^{t/10}$$

It's every 10 hours that we reduce the amount to 20% of what had been there.

So to remove 95% (which means leaving .05 times the original amount):

$$.05C_0 = C_0 (.2)^{t/10}$$

$$\ln .05 = \frac{t}{10} \cdot \ln .2$$

$$t = 10 \cdot \frac{\ln .05}{\ln .2} \approx 18.61 \text{ hours}$$

And to remove 99% (so leaving .01 times the original amount):

$$.01C_0 = C_0 (.2)^{t/10}$$

$$\ln .01 = \frac{t}{10} \cdot \ln .2$$

$$t = 10 \cdot \frac{\ln .01}{\ln .2} \approx 28.61 \text{ hours}$$

Wow! So it takes 10 hours longer.

But that makes sense: Because 1% is what's left when 80% of the 5% gets eliminated, these amounts should be 10 hours apart.