Calculus I Exam 1 Spring 1995 3/1/95

Each problem is worth 10 points. Be sure to show all work for full credit. Please circle all answers and keep your work as legible as possible. All answers must be reasonably simplified for full credit. Authorized personnel only beyond this point.

1. a) State the definition of the derivative of a function f.

b) If someone told you that the graph of some function f(x) is shaped like a parabola, opening upward, with its vertex at the origin, would you expect $f'(-1)$ to be positive or negative, and why?

2. The oil tanker S.S. Hazelwood has run aground and is leaking oil. If the amount of oil remaining in the Hazelwood's holds is given by the table below,

Time (hours)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
Oil (Mbl)	415	375	340	310	282	256	232

a) Estimate the rate at which oil is leaking 2 hours after the crash.

b) What is the sign of the second derivative 2 hours after the crash?

3. I left home, stopped at a stoplight, drove on, then realized I had left a yam in the oven at home. I drove back much faster than I had left, running several red lights along the way. Sketch graphs of my position, velocity, and acceleration, clearly indicating the times at which I stopped at the light and turned around.

4. If $f(x)=3/(x^2)$, use the definition of the derivative to find $f'(x)$.

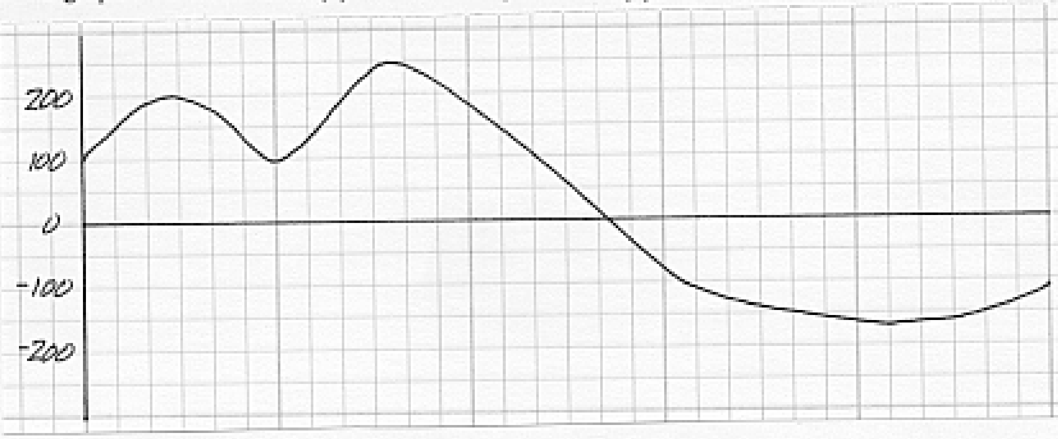
5. a) If $f(x)=x^{1/3}$, $f'(8)=1/12$. Use this fact to write an equation of the tangent line to $f(x)$ at the point $(8,2)$.

b) Use your linear approximation from part a to estimate the cube root of 8.1 (which is $f(8.1)$).

6. Given the following information about the derivatives of a function $f(x)$, sketch a rough graph of the function.

When x is:	$f'(x)$ is:	$f''(x)$ is:
less than 0	+	-
between 0 and 1	-	-
greater than 1	-	+

7. Given the graph of the function f(x) shown below, sketch f '(x) on the same set of axes.

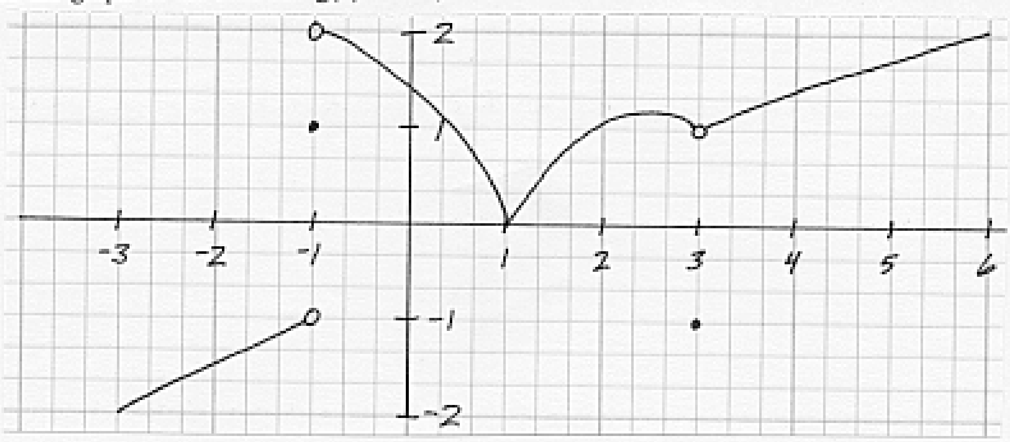


8. The Quabbin Reservoir in the western part of Massachusetts provides most of Boston's water. If the function f from problem 7 gives the rate of change of the amount of water in the reservoir during the month of July,

a) When was the water level increasing most quickly?

b) When was the water level highest?

9. Given the graph of the function g(x) below,



a) What is $\lim_{x \to -1} g(x)$, or why does it not exist?

b) For what values of x is the function not differentiable?

c) Explain why the function is or is not continuous when x=3.

10. A student is asked to estimate the slope of $f(x)=1/x$ when $x=½$. He decides to find slope of the secant line between $(½,f(½))$ and $(0,f(0))$. After working out the slope of this line, he declares that $f'(½)$ does not exist. Explain why he was right or wrong, and give a good approximation of $f'(½)$.

Extra Credit (5 points possible):
Use the definition of the derivative to find $f'(0)$ if $f(x)=\sin x$. Partial credit just for setting it up.
[Hint: the sum formula for sin, $\sin(a+b) = \sin a \cos b + \cos a \sin b$, and the fact that $\lim_{h\to 0}(\sin h)/h = 1$ might both be helpful.]