Exam 3 Calculus 1 12/5/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

1. Given the following table of data on the velocity (in feet per second) of the space shuttle t seconds after liftoff, give a low and a high estimate of the height of the shuttle after ten seconds. Explain why you think one estimate is high and the other low.

t	$v(t)$
0	0
2	33
4	68
6	108
8	147
10	185

$(2\cdot 0)+(2\cdot 33)+(2\cdot 68)+(2\cdot 108)+(2\cdot 147) = 712$ ft

$(2\cdot 33)+(2\cdot 68)+(2\cdot 108)+(2\cdot 147)+(2\cdot 185) = 1082$ ft

low estimate = 712 ft
high estimate = 1082 ft.

Good

The low estimate assumes the shuttle is going the speed at the start of the interval for the entire interval.
The high estimate assumes the shuttle is going the speed at the end of the interval for the entire interval

2. Find $\int \left(x^2 + \frac{1}{x} + \frac{1}{x^4} \right) dx$.

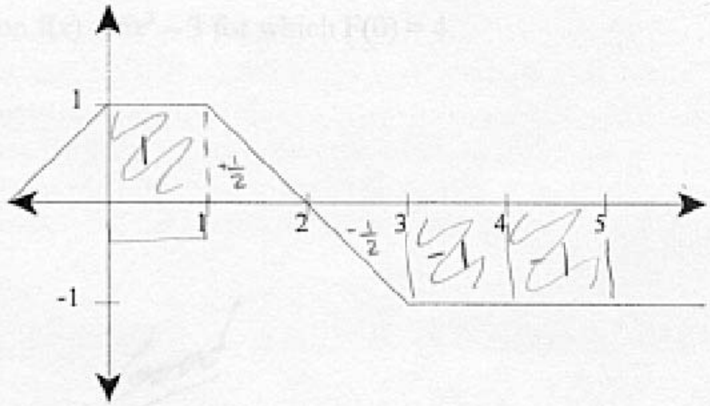
$\Rightarrow \int x^2 dx + \int \frac{1}{x} dx + \int x^{-4} dx$

$\Rightarrow \frac{1}{3}x^3 + \ln|x| + -\frac{1}{3}x^{-3}$

$\Rightarrow \frac{1}{3}x^3 + \ln|x| - \frac{1}{3\cdot x^3} + C$

Great

3. Given the graph of g(*x*) shown at right, what are:



(a) $\int_0^1 g(x)\,dx = \boxed{1}$

(b) $\int_0^5 g(x)\,dx = \boxed{-1}$ Great

4. If $F(x) = \int_3^x \sqrt{1+t^3}\,dt$, what is $F'(x)$?

$$F'(x) = \sqrt{1+x^3}$$

The Fundamental Theorem of Calculus says that the derivative of a function integrated from a constant to x will just be that function applied to x.

In short, the derivative and integral cancel out.

5. Find an antiderivative F(*x*) of the function $f(x) = 6x^2 - 3$ for which $F(0) = 4$.

$$f(x) = 6x^2 - 3$$

$$F(x) = \int f(x)\,dx$$

$$= \int (6x^2 - 3)\,dx$$

$$= \frac{6x^3}{3} - 3x + c$$

Now $F(0) = 4$

$$\therefore 4 = \frac{6\cdot(0)^3}{3} - 3\cdot 0 + c$$

$$\therefore c = 4$$

$$\boxed{F(x) = 2x^3 - 3x + 4}$$

Good

6. What is the **average value** of the function $g(x) = \sin x$ on the interval from $x = 0$ to $x = \pi$?

$$\frac{1}{\pi}\int_0^{\pi} \sin x = \frac{1}{\pi}\left[-\cos x\Big|_0^{\pi}\right. \quad -\cos\pi + +\cos 0$$

$$1 + 1$$

$$\boxed{\frac{1}{b-a}\int_0^x f(x)\,dx}$$

Avg. Value

Great

$$\frac{1}{\pi} \cdot 2 = \frac{2}{\pi}$$

7. Jake is a calculus student at a large university and he's frustrated. Jake says "This sucks so bad. Math is supposed to just be numbers, and our sucky book keeps asking things with words. I'm an engineering student, I'm not supposed to have to be able to read. So anyway, there's this question about this metal rod, and it's, like, bigger at one end than the other, so like more kilograms, right? And we're supposed to say what it means to do an integral of it from zero to two. What kind of crap is that? I just wanna work out an answer, not understand it. This touchy-feelie crap just pisses me off."

Explain clearly for Jake what the integral of a density function might tell us about a metal rod **and why.**

The integral of the density function will tell how much the metal rod weighs. If the density is in kilograms per foot and you take the integral with respect to distance (feet), the result will just be in kilograms. Also, if you divide this answer by the length of the rod, you will get the average density of the rod. Doing the integral from zero to two will get the total weight (mass?) in kilograms of the first 2 feet of the rod and diving by 2 will give you the average density of those two feet.

Excellent!

8. If $\int_0^b x^2\,dx = 2$, what is the exact value of b?

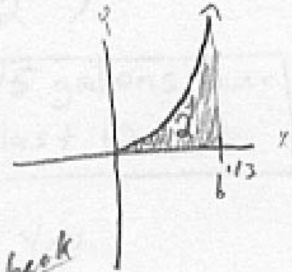
$$\frac{x^3}{3}\Big|_0^b = 2$$

$$\frac{b^3}{3} - \cancel{\frac{0}{3}} = 2$$

$$\frac{b^3}{3} = 2$$

$$b^3 = 6$$

$$\boxed{b = 6^{1/3}}$$



Check

$$\int_0^{6^{1/3}} x^2\,dx$$

$$\frac{x^3}{3}\Big|_0^{6^{1/3}}$$

$$\frac{(6^{1/3})^3}{3} - \cancel{\frac{0}{3}}$$

$$\frac{6}{3} = 2 \checkmark$$

Well done

9. Jon's sink has developed a drip. Ten days ago it was dripping at a rate of one gallon each day. Now it's dripping two gallons per day. Assuming that the rate of the dripping has increased linearly, how much water has dripped over the last ten days?

1 gal/day

2 gal/day $\rightarrow$ ten days Later

$\frac{2-1}{10} = 0.1$ gal/day increase per day for the past 10 days.

Low est: $(1)(1+1.1+1.2+1.3+1.4+1.5+1.6+1.7+1.8+1.9) = 14.5$ gal

High est: $(1)(1.1+1.2+1.3+1.4+1.5+1.6+1.7+1.8+1.9+2.0) = 15.5$ gal

Ave = 15 gallons $\downarrow$ # gallons dripped over 10 days.

Nice Job!

OR $\int_0^{10} 0.1x+1 = .05x^2\Big|_0^{10} + x\Big|_0^{10} = (.05)(100) + 10 = 15$ gallons

10. We worked out in class that if a car is traveling 88 ft/sec and brakes (with constant deceleration) to a stop in 4 seconds, then it travels 176 feet while coming to a stop.

(a) If instead the initial velocity is 110, but still can decelerate at 22 feet/sec^2, how far will the car travel before coming to a stop?

(b) If we just know that the car is traveling v_0 feet/sec when the brakes are first applied, but still can decelerate at 22 feet/sec^2, what distance (in terms of v_0) will it travel while coming to a stop?

$a = -22 \text{ ft/sec}^2$

$v = -22t + V_0$

$= -22t + 110$

$0 = -22t + 110$

$\frac{22t = 110}{22}$

$t = 5 \text{ sec}$ to stop

$S = -11t^2 + 110t$

$S(5) = -11(5)^2 + 110(5)$

$= -11(25) + 110(5)$

$= \underline{275 \text{ ft}}$

$a = -22 \text{ ft/sec}$

$v = -22t + V_0$

$0 = -22t + V_0$

$\frac{22t = V_0}{22}$

$t = \frac{V_0}{22}$

$S = -11t^2 + V_0 t$

$S(t) = -11\left(\frac{V_0}{22}\right)^2 + V_0\left(\frac{V_0}{22}\right)$

$= \frac{-11V_0^2}{484} + \frac{V_0^2}{22}$

$= \frac{-11V_0^2 + 22V_0^2}{484} = \underline{\frac{11V_0^2}{484} \text{ ft}}$

Nice!

it will have gone this far in terms of V_0