Calculus I **Exam 5** **Spring 1995** **5/5/95**

Each problem is worth 10 points. Be sure to show all work for full credit. Please circle all answers and keep your work as legible as possible. All answers must be reasonably simplified for full credit. Beware of Dog.

1. Find

$$x^{-3}$$

$$\int \left(x^4 - \frac{1}{x} + \frac{1}{x^3}\right) dx$$

$$\frac{x^5}{5} - \ln|x| + \frac{x^{-2}}{-2} + C$$

2. Find

$$\int_1^2 (x^3+1)\, dx$$

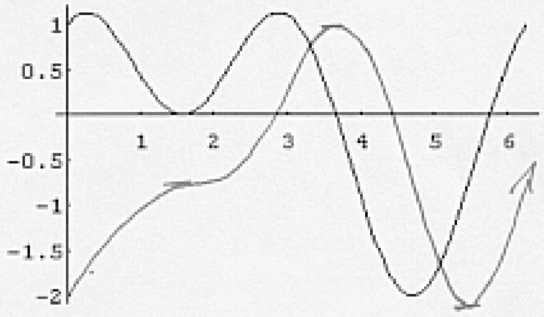
$$\frac{x^4}{4} + x \Big|_1^2$$

$$(4+2) - \left(\frac{1}{4}+1\right)$$

$$5 - \frac{1}{4}$$

$$\frac{19}{4}$$

3. Given the graph of g'(x) below, sketch a possible graph of g(x) on the same set of axes.



4. If a bowling ball's velocity in feet per second is given by v(t) = 8 - 0.1t and the bowling ball is 30 feet from the pins at time 0, write a function which gives the bowling ball's distance from the pins at time t.

$$d(t) = -8t + .05t^2 + 30$$

5. Wile E. Coyote has set up an explosive trap for the Roadrunner, but accidentally triggered it himself. The blast propells him straight up with an initial velocity of 50 meters per second. The acceleration due to gravity in cartoons is 5 meters per second^2. How long does it take him to peak, and how high does he go?

$a = -5$

$v = -5t + v_0$

$v(t) = -5t + 50$

$h(t) = -2.5t + 50t + h_0$

$h(t) = -2.5t + 50t$

Peaks when:

$0 = -5t + 50$

$5t = 50$

$t = 10$ seconds

Height when peaks:

$h(10) = -2.5(10)^2 + 50(10)$

$= -250 + 500$

$= 250$ meters

6. If you know

$$\int_0^2 (f(x)+g(x))\,dx=20, \quad \int_2^5 (f(x)+g(x))\,dx=8, \quad \int_0^5 2g(x)\,dx=10$$

$\int_0^5 g(x)\,dx = 5$

then what is

$$\int_5^0 f(x)\,dx = -23$$

$\int_0^5 f(x)+g(x) = 28$

$\int_0^5 f(x)dx + \int_0^5 g(x)dx = 28$

$\int_0^5 f(x)dx + 5 = 28$

$\int_0^5 f(x)dx = 23$

7. Find

$$\int \frac{x}{\sqrt{x^2+1}}\,dx$$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$= \int \frac{x}{u^{1/2}} \frac{du}{2x}$

$= \frac{1}{2}\int u^{-1/2} du$

$= \frac{1}{2} 2u^{1/2} + C$

$= (x^2+1)^{1/2} + C$

8. If Jon's Ford Festiva can go from 0 to 60 feet per second in six seconds flat, how much distance has it covered by the time it hits 60?

$\frac{60}{6} = 10$ ft/sec^2

$a = 10$

$v = 10t + v_0$

$v(t) = 10t$

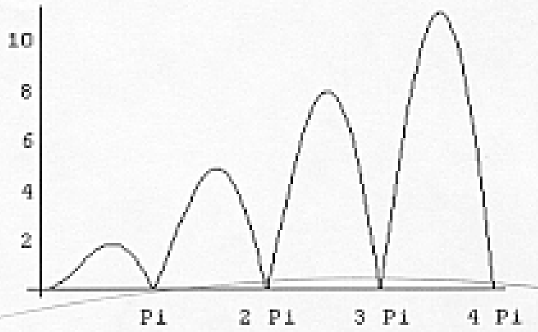
$d(t) = 5t^2 + d_0$

$d(t) = 5t^2$

$d(6) = 5(6)^2 = 5 \cdot 36 = 180$ ft

9. The graph of the function f(x) = |x sin x| is goofy (see below), but for positive values of x it's always between the graphs of g(x) = x and h(x) = 0. What can you say about

$$\int_0^{4\pi} |x \sin x|\,dx$$



$0 \le |x \sin x| \le x$

$\int_0^{4\pi} 0\,dx \le \int_0^{4\pi} |x \sin x|\,dx \le \int_0^{4\pi} x\,dx$

$0 \le \int_0^{4\pi} |x \sin x|\,dx \le \frac{x^2}{2}\Big|_0^{4\pi}$

$0 \le \int_0^{4\pi} |x \sin x|\,dx \le 8\pi^2$

It's greater than 0 but less than $8\pi^2$. ≈ 78.96

10. Biff, a Calc I student, is trying to find ∫ sec x dx. First he thinks it might be csc x, then changes his mind to tan x. Explain what might have led the Biffster to each of these ideas, and explain what's wrong or right about each.

Biff looked at $\int \frac{1}{\cos x}\,dx$ and just did the antiderivative of the bottom to get $\frac{1}{\sin x}$.
Then Biff remembered that the derivative of tan x is $\sec^2 x$ and jumped carelessly with that.
In both cases his guess doesn't work - its derivative isn't sec x.
Also he forgot the +C.

Extra Credit (5 points possible)
Just after peaking (in problem 5), the coyote switches on his Acme jetpack, which provides 10 meters per second² of thrust, but tragically it turns out to be strapped on backwards, propelling him straight to the ground. What is the last possible moment for him to flip over (so that the jetpack pushes him upward) in order to land with a velocity of 0?

~~[illegible]~~ Before Flip

$a = -15$

$v(t) = -15t$

$h(t) = -7.5t^2 + 250$

After Flip

$a = 5$

$v(t) = 5t - 15t_1$

$h(t) = 2.5t^2 - 15t_1 t + -7.5t_1^2 + 250$

$0 = 5t - 15t_1$

$3t_1 = t$

$0 = 2.5(3t_1)^2 - 15t_1(3t_1) - 7.5t_1^2 + 250$

$0 = 22.5t_1^2 - 45t_1^2 - 7.5t_1^2 + 250$

$30t_1^2 = 250$

$t_1^2 = \frac{25}{3}$

$t_1 = \sqrt{\frac{25}{3}} \approx 2.88675134595$ seconds after peak