Exam 1   Calculus 3   9/12/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Everything in the book is true, but not everything true is in the book.

1. Write a vector equation for the line passing through the points (-5,7,3) and (1,2,4).

2. A koala bear is sitting in a tree at the juncture of two branches with directions given by the vectors $3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} - 5\mathbf{j} + \mathbf{k}$. Find the angle between the branches.

3. Find the length of the curve $\mathbf{r}(\mathrm{t}) = \langle 2 \sin t, 5t, 2 \cos t \rangle$ for $-10 \leq t \leq 10$.

4. Find the curvature of $\mathbf{r}(\mathrm{t}) = \langle t, t^2, t^3 \rangle$ (which is a curve called a *twisted cubic*, which is a pretty funny name) at the point (0,0,0).

5. Convert the point with cylindrical coordinates $(3, 3\pi/4, 2)$ to

a) Rectangular coordinates

b) Spherical coordinates

6. Write an equation for the plane passing through the point $(0,0,c)$ and containing the vectors $\langle 1,0,m \rangle$ and $\langle 0,1,n \rangle$.

7. Show that for any three-dimensional vectors **a** and **b**, **a** × **b** must be perpendicular to **a**.

8. Biff is a Calc 3 student at a large state university, and he's having some trouble. Biff says "Man, this calc stuff is kicking my butt. There was this problem on our test, and there were four points, and you were supposed to say if they all were on the same line or not, like in three dimensions. I could totally do it if there were three points, 'cause there was an example like that in the book, but they never told us how to do it if there was four points. It's totally unfair, when they haven't even told us how to do it yet. I wrote that it was impossible to determine it if there were four points, because I figured if there were a way, then it would have been in the book, right?"

Explain clearly to Biff a reasonable way to determine whether four points all lie on a common line, or defend his assertion that it's impossible to determine whether such a thing happens.

9. Prove or give a counterexample: The dot product is *associative*, i.e. for any three three-dimensional vectors **a**, **b** and **c**,

$$\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$$

10. Suppose you want to write an equation for a hyperboloid of one sheet. You want it to open along the z-axis, and have its smallest cross section, a circle with radius 3, where it crosses the plane $z = 1$. You also want it to have a circle with radius 5 for its cross section where it crosses the plane $z = 0$. Find an equation for such a surface.

Extra Credit (5 points possible): A cable has radius r and length L and is wound around a spool with radius R without overlapping. What is the shortest length along the spool that is covered by the cable?