Exam 3   Calculus 3   10/29/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

$$\boxed{x = \rho \sin\phi \cos\theta \quad y = \rho \sin\phi \sin\theta \quad z = \rho \cos\phi}$$

1. If the function $h(x, y) = -2x + 3y + 2000$ gives the density of field mice per square mile in Kansas (where Kansas is taken to be a rectangle with its lower left corner at the origin and other vertices at (400,0), (400,200), and (0,200)), **write an integral** for the total number of field mice in Kansas.

2. **Set up** an iterated integral for the area of the region inside $x^2 + y^2 = 9$, outside $x^2 + y^2 = 4$, above $y = 0$, and below $y = x$.

3. **Set up** iterated integrals for $\bar{z}$, the z coordinate of the center of mass of the first-octant portion of a sphere with radius 3 and uniform density k.

4. **Set up** an iterated integral for the volume of the tetrahedron with vertices (0,0,0), (2,0,0), (0,3,0), and (0,0,6).

5. **Set up** an iterated integral for the **surface area** of the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 9$.

6. Evaluate $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{4 + x^3}\, dx\, dy$ **exactly**.

7. **Compute the Jacobian** of the transformation $x = \frac{1}{3}(u+v), y = \frac{1}{3}(v-2u)$.

8. Biff is having some trouble with iterated integrals. Biff says "Man, we had this quiz and I know I did it wrong, 'cause I worked out this double integral and got zero. Volume can't be zero, so I must have screwed up, but I went over it twenty times and I have no idea what was wrong. It wasn't that complicated, either, the thing we integrated was just x, so I don't know how I messed up."

Explain clearly to Biff whether zero is automatically a wrong answer for a double integral where the integrand is x, and why.

9. Evaluate $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} 2\,dz\,dy\,dx$ **exactly**.

10. **Set up** an iterated integral to find the volume of the solid in the first octant bounded by the elliptic cylinder $y^2 + 4z^2 = 4$ and the plane $y = x$.

Extra Credit (5 points possible):

Set up an iterated integral and use it to find the surface area of a sphere with radius R.