Exam 4a Calculus 3 11/25/2003

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F}(x,y) = \langle 6xy, 3x^2 + 3y^2 \rangle$ and C is the first-quadrant portion of a circle (centered at the origin) from (3,0) to (0,3).

2. If $\mathbf{F}(x,y,x) = \langle 3, 5xy, z^2 \rangle$ and S is the half of a sphere of radius 5 centered at the origin for which $x \geq 0$, **say what line integral you could work out instead** which is equivalent to the surface integral $\iint_S \operatorname{curl}\mathbf{F} \cdot d\mathbf{S}$, and parametrize the path involved (but you don't need to work it out entirely).

3. Compute $\nabla\times\mathbf{F}$ for $\mathbf{F}(x,y,z) = \langle 3x,0,3y\rangle$

4. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = 3xy\mathbf{i} + 6x^2\mathbf{j}$ and C is the path consisting of four line segments joining the points (0,0), (3,0), (3,2), and (0,2) in that order.

5. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = 2x\mathbf{i} + 2y\mathbf{j} + \mathbf{k}$ and S is the portion of $z = x^2 + y^2$ between the planes $z = 1$ and $z = 9$.

6. Prove that if $f(x,y,z)$ is a function with continuous second-order partial derivatives, then $\mathrm{curl}(\nabla f) = \mathbf{0}$. Make it clear how the requirement that the partials be continuous is important.

7. Bunny is a calc 3 student at a large state university and she's having some trouble. Bunny says "Ohmygod, I am so totally confused by this class. I mean, I can work out a lot of the problems, but I totally don't understand what any of it means. I guess it mostly doesn't really matter, since our exams are all multiple choice, but there was this one on the old exam I got from the files in my boyfriend's frat where, like, it was a surface integral, and since they set it up by parametrizing the surface a totally different way than I did, their setup looked different than mine. It was multiple choice, and you were only supposed to set it up, so since my setup was different from theirs I totally didn't know which answer to mark. How can you tell?"

Explain as clearly as possible to Bunny what relationships might hold between surface integrals set up using different parametrizations of the same portion of the same surface.

8. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \langle -x, y \rangle$ and C is a circular path (centered at the origin) beginning at (1,0) and traversing n quarter-circles (where, for instance, traversing 8 quarter-circles means passing twice around a circle).

9. Suppose that **F** is a vector field whose divergence is zero everywhere and S is a tetrahedron consisting of the four sides S_1, S_2, S_3, and S_4. If you know that $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} = 2$, $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S} = 2$, and $\iint_{S_3} \mathbf{F} \cdot d\mathbf{S} = 2$, then what can be said about $\iint_{S_4} \mathbf{F} \cdot d\mathbf{S}$? Explain your reasoning.

10. We talked in class about checking to see if a vector field $\mathbf{F}(x,y) = <P(x,y),Q(x,y)>$ is conservative by comparing $P_y(x,y)$ and $Q_x(x,y)$. The analogous question for three-dimensional vector fields is a bit more involved: How could you test whether a vector field $\mathbf{F}(x,y,z) = P(x,y,z),Q(x,y,z),R(x,y,z)>$ is conservative (apart from just seeking a potential function)?

Extra Credit (5 points possible):

Compute $\displaystyle\int_C \left\langle \frac{x}{x^2+y^2}, \frac{-y}{x^2+y^2} \right\rangle \cdot d\mathbf{r}$ for C the counterclockwise circle centered at the origin of radius R.