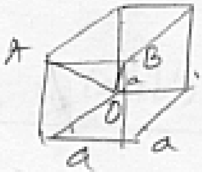
Problem Set 1 $\frac{20}{20}$

1. A cube with side a.

$\overline{OA} = \langle 0, a, a \rangle$

$\overline{OB} = \langle a, a, a \rangle$.

Nice



$\overline{OB}^2 = a^2 + \sqrt{2}a^2$

$= 3a^2$

$OB = \sqrt{3}\,a$.

$\overline{OA} \cdot \overline{OB} = |\overline{OA}|\,|\overline{OB}| \cos\theta$

$a^2 + a^2 = \sqrt{a^2+a^2}\,\sqrt{a^2+a^2+a^2}\,\cos\theta$.

$\frac{2a^2}{\sqrt{2}a \cdot \sqrt{3}a} = \cos\theta$.

$\sqrt{\frac{2}{3}} = \cos\theta$

$\theta = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

(39) 2. $\vec{a} \cdot \vec{b} = \langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle$

$= a_1 b_1 + a_2 b_2 + a_3 b_3$

$= b_1 a_1 + b_2 a_2 + b_3 a_3$ Good

$= \langle b_1, b_2, b_3 \rangle \cdot \langle a_1, a_2, a_3 \rangle$

$= \vec{b} \cdot \vec{a}$

4. $(c\vec{a}) \cdot \vec{b} = c\langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle$

$= \langle ca_1, ca_2, ca_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle$

$= ca_1 b_1 + ca_2 b_2 + ca_3 b_3$

$= c(a_1 b_1 + a_2 b_2 + a_3 b_3)$

$= c(\langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle)$

$* = c(\vec{a} \cdot \vec{b})$

$= c(\langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle)$

$= c(a_1 b_1 + a_2 b_2 + a_3 b_3)$

$= ca_1 b_1 + ca_2 b_2 + ca_3 b_3$

$= a_1 cb_1 + a_2 cb_2 + a_3 cb_3$ Nice

$= \langle a_1, a_2, a_3 \rangle \cdot (c\langle b_1, b_2, b_3 \rangle)$

$* = \vec{a} \cdot (c\vec{b})$

5. $0 \cdot \vec{a} = 0 \cdot \langle a_1, a_2, a_3 \rangle$

$= 0(a_1) + 0(a_2) + 0(a_3)$

$= 0 + 0 + 0$

$= 0$

22 4. $(\vec{a}+\vec{b}) \times \vec{c} = \langle a_1+b_1, a_2+b_2, a_3+b_3 \rangle \times \vec{c}$

$$= \langle a_1+b_1, a_2+b_2, a_3+b_3 \rangle \times \langle c_1, c_2, c_3 \rangle$$

$$= \langle (a_2+b_2)c_3 - (a_3+b_3)c_2, (a_3+b_3)c_1 - (a_1+b_1)c_3, (a_1+b_1)c_2 - (a_2+b_2)c_1 \rangle$$

$$= \langle (a_2c_3 + b_2c_3) + (-a_3c_2 - b_3c_2), (a_3c_1 + b_3c_1) + (-a_1c_3 - b_1c_3), (a_1c_2 + b_1c_2) + (-a_2c_1 - b_2c_1) \rangle$$

$$= \langle (a_2c_3 - a_3c_2) + (b_2c_3 - b_3c_2), (a_3c_1 - a_1c_3) + (b_3c_1 - b_1c_3), (a_1c_2 - a_2c_1) + (b_1c_2 - b_2c_1) \rangle$$

$$= \langle a_2c_3 - a_3c_2, a_3c_1 - a_1c_3, a_1c_2 - a_2c_1 \rangle + \langle b_2c_3 - b_3c_2, b_3c_1 - b_1c_3, b_1c_2 - b_2c_1 \rangle$$

$$= \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$$

Well done

12.4 #41 Prove $(a-b)\times(a+b)=2(a\times b)$

Using Theorem 8 part 4

$= (a-b)\times(a+b) = \underline{a\times(a+b)} - \underline{b\times(a+b)} = \underline{a\times a} + \underline{a\times b} - \underline{b\times a} - \underline{b\times b}$

Since crossing a vector with itself is <u>zero</u> then $a\times a=0$ and $b\times b=0$

$(a-b)\times(a+b) = 0 + a\times b - b\times a - 0$

Using Theorem 8 part 1 $\underline{-b\times a = a\times b}$

$= a\times b - b\times a = a\times b + a\times b = \underline{2(a\times b)}$

Well done