Quiz 3 Calculus 3 11/11/03

Each problem is worth 5 points. For full credit indicate clearly how you reached your answer.

1. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F}(x,y) = \langle 6xy, x+y\rangle$ and C is the line segment from (2,0) to (3,5).

1. $x(t) = 2+t$
   $y(t) = 5t$ $\quad r(t) = \langle 2+t, 5t\rangle$
   $0 \le t \le 1$

2. $F(r(t)) = \langle 6(2+t)(5t), (2+t)+5t\rangle$
   $6(10t+5t^2), 2+6t$
   $\langle 60t+30t^2, 2+6t\rangle$

3. $r'(t) = \langle 1,5\rangle$

4. $\int_C \langle 60t+30t^2, 2+6t\rangle \cdot \langle 1,5\rangle\, dt$

5. $\int_0^1 (60t+30t^2+10+30t)\, dt$
   $\int_0^1 (90t+30t^2+10)\, dt$
   $45t^2+10t^3+10t \Big]_0^1 = 45+10+10 = \boxed{65}$

Great.

2. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F}(x,y) = \langle 6xy, 3x^2+2y\rangle$ and C is the top half of a circle (centered at the origin) from (3,0) to (-3,0).

Use Fund. Theorem for Line Integrals!
Clairvant says yes we can!

$f_{xy} = 6x \quad f_{yx} = 6x$

$f(x,y) = 3x^2y + y^2$

$\int \vec{F}\cdot d\vec{r} = f(-3,0) - f(3,0)$
$= [3(-3)^2\cdot 0 + 0^2] - [3(3^2)0 - 0^2]$
$= 0 - 0$
$= 0$

Excellent