Exam 1 Calc 2 9/10/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\int \sin t \, dt$.

2. Find the area below the curve $f(x) = 3/x^2$ but above the x axis between $x = 1$ and $x = 2$.

3. Evaluate $\int \frac{y}{y^2+4}\,dy$.

4. Evaluate $\int \frac{1}{t^2+4t+5}\,dt$.

5. If $F(x) = \int_{1}^{x} \frac{1}{\sqrt{1+t^3}}\, dt$,

a) What is F(1) and why?

b) What is $F'(2)$ and why?

6. Use the trig substitution $x = 2\sin\theta$ to show that the integral $\int x^2\sqrt{4-x^2}\,dx$ is equivalent to the integral $16\int \sin^2\theta\cos^2\theta\,d\theta$.

7. To celebrate Halloween, Jon plans to launch pumpkins across the quad from his office window (which is 16 feet above the ground). He wants the pumpkins to reach their maximum height after 2 seconds of flight.

a) What initial vertical velocity should the pumpkin be launched with to achieve this?

b) What will the pumpkin's velocity be when it hits the ground?

8. Biff is a calculus student at a large state university, and he's having some trouble. Biff says "Man, calculus sucks. This integral stuff just makes no sense. I mean, there's left-handed stuff and right-handed stuff and middle-handed stuff and all that, and it's just crazy. There was this question on our quiz, like they gave us this function with the square root of this one plus x to the fourth stuff, and we had to integrate it from zero to two, and it asked if these different estimates was too high or too low or whatever. But I don't have one of those fancy calculators that integrates, so there's like no way I can know if those estimates are too high or too low, right? Am I just supposed to guess? What kinda crap is that?"

Explain clearly to Biff how he could know whether to expect the left, right, and midpoint approximations to be high or low without needing to know the true value of the integral.

9. Show how the integration formula

$$\int e^{ax} \sin(bx)\, dx = \frac{1}{a^2+b^2} e^{ax} \left[a\sin(bx) - b\cos(bx) \right] + C$$

can be obtained using the techniques of integration we know.

10. Our table of integrals doesn't have a formula for $\int \frac{1}{(x-a)(x-b)(x-c)}\,dx$. Show how one can be found using the techniques of integration we know.

Extra Credit (5 points possible):
Express with definite integrals the **area** between $x = 0$ and $x = 2$, bounded by a function of the form $f(x) = x^2 - b$ and the line $y = 0$. For what value of the constant b will this area be least?