Exam 2 Calc 2 9/10/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **Set up** an integral for the present value of an income stream of $p(t) = 50000 - 5000t$ over the next ten years.

2. Suppose that 6J of work are required to stretched a spring to a length of 60cm from its natural length of 50cm. How much work is required to stretch it from 50cm to 80cm?

3. **Set up** an integral for the area of the region between $x = 5y - y^2$ and $y = x$.

4. **Set up** an integral for the length of the curve $y = x^2$ between the points (-1,1) and (3, 9).

5. Evaluate $\int_{1}^{4} \frac{2}{(x-3)^2}\,dx$.

6. Suppose that within some Central American district the probability of a household having an income of d dollars/year is given by $p(d) = \frac{1}{2000} - \frac{d}{8000000}$ for values of d between 0 and the district's maximum household income. **Write an equation** which, when solved for b, gives the median household income in this district.

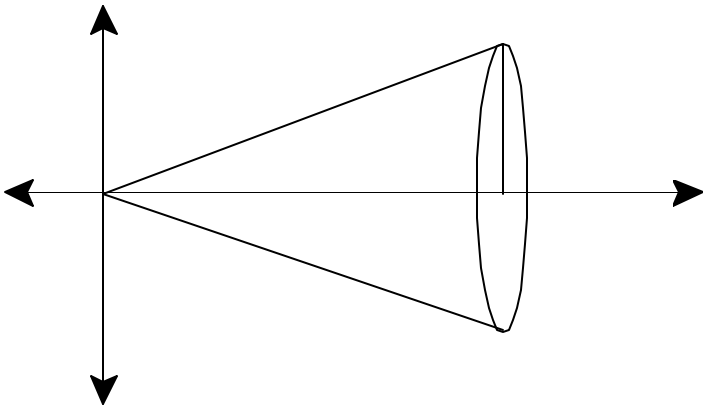
7. **Set up** integrals for the x-coordinate of the center of mass of the first-quadrant portion of a circle with radius r.

8. Bunny is a calculus student at a large state university, and she's having some trouble. Bunny says "Ohmygod, this calculus stuff is sooooo freaky. I mean, there was this stuff in class today, like where you find the area of stuff that goes to infinity, right? And it just makes no sense at all, 'cause everybody knows you do area with length times width, and so if length is to infinity then automatically area is infinity too, right? So there's just no way. They all should be infinity, and then on the test I guess I'll just say that for all of them, because it has to be right.

Explain clearly to Bunny why the values of some improper integrals should be something other than infinity, even though the regions they pertain to have an infinite length or width.

9. The horse that lives in the pasture next door to Jon's house is leaning over the fence and drinking from the birdbath in Jon's garden. The basin of the birdbath is shaped like the solid of revolution formed by taking the region above $y = x^2/144$ and below $y = 1$. If the basin starts out full of water, and the water all gets slurped up to the back of the horse's throat, 12 inches above the bottom of the birdbath, **set up** an integral for the amount of work the horse does in the process.

10. A cone can be obtained by revolving the region below a certain line around the *x* axis, as suggested in the picture below. **Set up** an integral for the volume of a cone with height *h* and base radius *r* **and use it** to show that the volume of such a cone is $V = \frac{1}{3}\pi r^2 h$.



Extra Credit (5 points possible):

The integral $2p\int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$ gives the surface area of the solid obtained by revolving the region between f(x) and the x axis between x = a and x = b around the x axis. Find the surface area

of the cone from problem 10.