Exam 3 Calc 2 11/5/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **Find the sum** of the series $4 - 1 + 1/4 - 1/16 + 1/64 - \ldots$

2. Write the series $-\frac{1}{3} + \frac{2}{5} - \frac{3}{7} + \frac{4}{9} - \frac{5}{11} + \ldots$ in sigma notation.

3. Find the Taylor series for sin x of degree 4 centered at $a = \pi/2$.

4. Find the first 5 non-zero terms of the MacLaurin series for $\mathrm{f}(x) = e^{\left(x^3\right)}$.

5. Find the **radius of convergence** of the series $\sum_{n=0}^{\infty} \frac{(3x)^n}{2n+1}$.

6. Use a series of degree at least 5 to **approximate ln 2**.

7. Biff is a calculus student at a large state university, and he's having some trouble. Biff says "Man, this series stuff is killing me. Our professor gave us, like, this study sheet with a bunch of problems that might be on our test, like thirty of 'em, and he's gonna pick just some of 'em for the test. He said it's multiple choice, just say if it converges or if it diverges. So one of the problems was for the series one over x cubed, and I think he screwed up, 'cause that one doesn't converge or diverge, it's neither, 'cause the ratio test says one for it, so that means it doesn't converge or diverge, right? So I guess the exam really has to be with three choices, like 'converge', or 'diverge', or 'can't tell', right?"

Explain clearly to Biff whether what he's saying is true or false, and why.

8. Determine whether x = -5 is included in the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{5^n \cdot n}$.

9. Determine whether the series $\sum_{n=0}^{\infty} \frac{2^n}{n!+1}$ converges or diverges. Be very clear in your justification.

10. Show how a Taylor polynomial can be used to find $\lim_{x \to 0} \frac{\sin 3x}{x}$.

Extra Credit (5 points possible):

Find MacLaurin Series of degree 7 for the following functions:

(a) $e^{i\theta}$

(b) $\cos \theta$

(c) $i \cdot \sin \theta$