

**Problem Set 2      Calculus 2      Due 9/17/2004**

You are encouraged to work in groups of two to four on this assignment and make a single group submission. Each problem is worth 5 points. For full credit indicate clearly how you reached your answer. All work must be legible and submitted on clean paper without ragged edges.

1. a) Find  $\int_0^{2\pi} \sin x dx$ .

b) Find  $\int_0^{20\pi} \sin x dx$ .

c) Find  $\int_0^{200\pi} \sin x dx$ .

d) Explain why the pattern of answers in parts a-c might lead someone to a wrong value for the integral  $\int_0^{\infty} \sin x dx$ . What should be said about the value of this improper integral?

2. Consider the integral  $\int_1^{\infty} \frac{\sin x}{x^2} dx$ . We can't find the antiderivative necessary to evaluate this

integral directly, but show how comparison to other integrals can let us at least determine whether this integral converges or not.

3. The gamma function is defined for all  $x > 0$  by the rule  $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$ .

a) Find  $\Gamma(1)$ .

b) Find  $\Gamma(2)$ .

c) Find  $\Gamma(3)$ .

d) Find  $\Gamma(4)$ .

e) Find  $\Gamma(5)$ .

f) Show that  $\Gamma(n + 1) = n\Gamma(n)$  for all positive  $n$ .

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