

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{-1}{3}\right)^n$.

$$a = \left(\frac{-1}{3}\right)^1 = \left(\frac{-1}{3}\right)$$

$$r = \frac{-1}{3}$$

$$\left(\frac{-1}{3}\right)^1 + \left(\frac{-1}{3}\right)^2$$

$$-\frac{1}{3} + \frac{1}{9}$$

$$\frac{\frac{1}{9}}{-\frac{1}{3}} = -\frac{1}{3}$$

$$S = \frac{a}{1-r}$$

$$S = \frac{-\frac{1}{3}}{1 - (-\frac{1}{3})}$$

$$S = \boxed{-\frac{1}{4}}$$

Excellent

2. Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges or diverges.

Integral Test

$$\lim_{x \rightarrow \infty} \int_2^x \frac{1}{x(\ln x)^2} dx =$$

$$\begin{aligned} \text{let } u &= \ln x \\ \frac{du}{dx} &= \frac{1}{x} \\ du \cdot x &= dx \end{aligned}$$

$$\lim_{x \rightarrow \infty} \int_2^x \frac{1}{x(u)^2} du \cdot x$$

$$\lim_{x \rightarrow \infty} -u^{-1} \Big|_{x=2}^{x=x} =$$

$$\lim_{x \rightarrow \infty} \frac{-1}{\ln x} \Big|_2^x \quad 0 - -\frac{1}{\ln 2}$$

Well done

converges

$\ln x$ gets bigger so
or $\ln \infty$

$\frac{1}{\ln x}$ goes to zero Yes.
or $\frac{1}{\ln \infty}$