Exam 1 Calc 3 9/24/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of f(x, y) with respect to y.

2. Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^3 + y^3}$ does not exist. Be clear about the reason.

3. Find the gradient of the function $f(x, y) = (x + y)e^y$.

4. If $w = f(x, y, z)$, $x = g_1(u, v)$, $y = g_2(u, v)$, and $z = g_3(u, v)$, state the appropriate form of the chain rule for $\dfrac{\partial z}{\partial v}$. Be sure to indicate clearly whether each derivative is a partial or regular.

5. If $f(x, y) = \ln(ye^{xy})$, find the directional derivative of f in the direction of the vector $\vec{u} = \dfrac{\left(5\vec{i} - 12\vec{j}\right)}{13}$.

6. Prove that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Biff is a calculus student at a large state university, and he's having some trouble. Biff says "Man, this stuff is hard. There was this question on our test about, like, if the level curves of this thing were all spaced out evenly, what did that mean, right? So I said, like, it meant the thing was a plane, 'cause the professor said something like that in class I think. But I got it back with 1 point, and it turns out this guy doesn't even grade on a curve. That curve was the only thing that got me through Calc 2, what the heck am I going to do now?"

Explain to Biff what conclusion you can draw if you know a surface's level curves are evenly spaced, and why.

8. Write an equation for the plane with normal vector $\vec{n} = a\vec{i} + b\vec{j} + c\vec{k}$ passing through the origin. Considering the plane as a function of x and y, in which direction will its directional derivative be greatest, and how large will the directional derivative be in that direction?

9. Find all points on the surface $z = x^2 + y^2 + 2$ whose tangent planes pass through the origin.

10. Find a function $f(x, y, z)$ for which $f_x = 2x - 3z$, $f_y = -6y^2z$, and $f_z = 3x - 2y^3 + 5$, or explain why one cannot exist.

Extra Credit (5 points possible):
Two surfaces are called *orthogonal* at a point of intersection if their normal vectors are perpendicular at that point. If S_1 is a sphere with radius 1 centered at the origin and S_2 is a sphere of radius 2 centered at the point $(0, 0, c)$, what should the value of c be in order for the two spheres to be orthogonal?