Exam 3 Calc 3 12/3/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

$$\boxed{\int_C \langle P, Q\rangle \cdot d\vec{r} = \iint_D \left(\frac{\delta Q}{\delta x} - \frac{\delta P}{\delta y} \right) dA}$$

1. Compute the divergence of the vector field $\vec{F} = 2xy\vec{i} + y^2\,\vec{j} - 3\vec{k}$.

2. Compute the curl of the vector field $\vec{F} = 2xy\vec{i} + y^2\,\vec{j} - 3\vec{k}$.

3. Evaluate $\int_C \langle -\sin y \sin x, \cos y \cos x \rangle \cdot d\vec{r}$ where C is the line segment from (π/2, 0) to (0, π/2)

4. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \langle -y, x \rangle$ and C is the right half of a circle of radius 5 centered at the origin, traversed counterclockwise, then followed by the line segment from (0, 5) to (0, -5).

5. Let *a*, *b*, and *c* be constants. Compute $\iint\limits_S \langle a, b, c\rangle \cdot d\mathbf{S}$ for *S* the surface of a sphere with radius 3 centered at the origin.

6. If $\vec{F}$ is any vector field whose components have continuous second partial derivatives, show that div curl $\vec{F}$ = 0.

7. Biff is a calculus student at a large state university, and he's having some trouble. Biff says "Dude, this is crazy. Our professor is psychotic. He gave us this test that was so long I don't think anybody could do it in an hour. But one of the problems was totally beyond all the others, 'cause it had, like, these five different surfaces, like a cone, a paraboloid, the top half of a sphere, the bottom half of a sphere, and something else I can't even remember, and we were supposed to show their flux integrals were all equal. I mean, just doing five flux integrals would take anybody more than an hour! But to top it off he had the vector field was one where we had to do its curl too. The man has totally lost his mind. A bunch of us were talking about, like, calling the psych department and having them study him or something."

Explain to Biff, in terms he can understand, how the professor might have actually intended the problem mentioned to be possible.

8. Compute $\int_S \left\langle x^2, 2y^2, 3z^2 \right\rangle \cdot d\vec{A}$ where S is the square with vertices (1,0,1), (2,0,1), (2,1,1), and (1,1,1).

9. Let C be a line segment from the point $(g_3(a), a)$ to the point $(g_1(a), a)$, and let $\vec{F}(x,y) = \langle 0, Q(x,y) \rangle$ for some function $Q(x, y)$. Compute $\int_C \vec{F} \cdot d\vec{r}$.

10. Suppose that C_1 is a line segment beginning at (x_1, y_1) and ending at (x_2, y_2), and that $\int_{C_1} \langle 4x, 2y \rangle \cdot d\vec{r} = 1$. How many paths C_n can exist which also begin at (x_1, y_1) and end at (x_2, y_2), but for which $\int_{C_n} \langle 4x, 2y \rangle \cdot d\vec{r} = 2$? Why?

Extra Credit (5 points possible):

Describe the collection of all points (x, y) such that $\int_{C} \langle 4x, 2y \rangle \cdot d\vec{r} = 1$, where C is the line segment from (x, y) to (x_2, y_2).