Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find all critical points of the function \( f(x, y) = x^3 + y^2 - 3x^2 + 10y + 6 \).

\[
\begin{align*}
\frac{f_x}{3x^2 - 6x} &= 0 \\
\frac{f_y}{2y + 10} &= 0 \\
\frac{f_{xx}}{3x(x - 2)} &= -10 = 2y \\
x &= 0 \text{ or } 2
\end{align*}
\]

Critical points are \((0, -5)\) and \((2, -5)\) when the partial derivatives equal zero there is a critical point.

Excellent!

2. The function \( g(x, y) = 2x^3 + xy^2 + 5x^2 + y^2 \) has one of its critical points at \((-1, 2)\). Classify that critical point as a local minimum, local maximum, or saddle point.

\[
\begin{align*}
g_x(x, y) &= 6x^2 + y^2 + 10x \\
g_y(x, y) &= 2xy + 2y \\
g_{xx}(x, y) &= 12x + 10 \\
g_{yy}(x, y) &= 2x + 2 \\
g_{xy}(x, y) &= 2y
\end{align*}
\]

\[
D(-1, 2) = g_{xx} \cdot g_{yy} - [g_{xy}]^2
\]

\[
D(-1, 2) = \left[12(-1) + 10\right] \cdot \left[2(-1) + 2\right] - [2(2)]^2
\]

\[
-12 + 10 \quad -2 + 2 \quad -4^2
\]

\[
-2 \quad 0 \quad -16
\]

\[
0 - 16
\]

\[
D < 0 \text{ so therefore it is a SF.}
\]