

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find all critical points of the function $f(x,y) = x^3 + y^2 - 3x^2 + 10y + 6$.

$$f_x = 3x^2 - 6x$$

$$0 = 3x^2 - 6x$$

$$0 = 3x(x-2)$$

$$x = 0 \text{ or } 2$$

$$f_y = 2y + 10$$

$$0 = 2y + 10$$

$$-10 = 2y$$

$$y = -5$$

when the partial derivatives equal zero there is a critical point.

Excellent!

critical points are
(0, -5) and (2, -5)

2. The function $g(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$ has one of its critical points at $(-1, 2)$. Classify that critical point as a local minimum, local maximum, or saddle point.

$$g_x(x,y) = 6x^2 + y^2 + 10x$$

$$g_y(x,y) = 2xy + 2y$$

C.P. is $(-1, 2)$

$$g_{xx}(x,y) = 12x + 10$$

$$g_{yy}(x,y) = 2x + 2$$

$$g_{xy}(x,y) = 2y$$

$$D(-1,2) = g_{xx} \cdot g_{yy} - [g_{xy}]^2$$

$$D(-1,2) = [12(-1) + 10] \cdot [2(-1) + 2] - [2(2)]^2$$

$$-12 + 10$$

$$-2 + 2$$

$$- 4^2$$

$$-2$$

$$\cdot$$

$$0$$

$$-$$

$$16$$

$$0$$

$$-16$$

Nice Job!

$D < 0$ so therefore it is a S.P.