

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Compute $\int_C \langle 2xe^y + 3x^2 + 2y, x^2e^y + 2x \rangle \cdot d\vec{r}$ for a path consisting of the first-quadrant portion of a circle (centered at the origin) of radius 5, traversed counterclockwise.

$$\therefore \frac{\partial}{\partial x} (x^2e^y + x^3 + 2xy) = 2xe^y + 3x^2 + 2y$$

$$\frac{\partial}{\partial y} (x^2e^y + x^3 + 2xy) = x^2e^y + 2x$$

$\therefore x^2e^y + x^3 + 2xy$ is the potential function
and the starting point is (5, 0) and the ending point is (0, 5)

$$\therefore [x^2e^y + x^3 + 2xy]_{(5,0)}^{(0,5)} = -(25 + 125) = \underline{-150} \quad \text{Excellent}$$

2. Compute $\int_C \langle xy, 2y \rangle \cdot d\vec{r}$, where C is the line segment from (0, 2) to the origin (0, 0)

$f_{xy} = x \neq 0 = f_{yx} \rightarrow$ there is no potential function

1. parametrize

$$x(t) = 0 +$$

$$y(t) = -2t + 2$$

$$\vec{F}(t) = \langle 0, -2t + 2 \rangle$$

for $0 \leq t \leq 1$

$$2. \vec{F}(\vec{r}(t)) = \langle 0(-2t+2), 2(-2t+2) \rangle$$

$$= \langle 0, -4t + 4 \rangle$$

$$3. \vec{r}'(t) = \langle 0, -2 \rangle$$

$$4. \int_0^1 \langle 0, -4t + 4 \rangle \cdot \langle 0, -2 \rangle dt$$

$$\int_0^1 0 + (8t - 8) dt$$

$$\int_0^1 (8t - 8) dt$$

$$4t^2 - 8t \Big|_0^1$$

$$4(1)^2 - 8(1) - 0$$

$$4 - 8 = \underline{-4}$$

Nice