Quiz

Let $\vec{F}(x,y,z) = a\vec{i} + b\vec{j} + c\vec{k} = \langle a, b, c \rangle$

Let S be a sphere w/ $r=1$

Find $\int_S \vec{F} \cdot d\vec{S}$

I. $x(u,v) = \cos u \sin v$

$y(u,v) = \sin u \sin v$

$z(u,v) = \cos v$

$\vec{r}(u,v) = \langle \cos u \sin v, \sin u \sin v, \cos v \rangle$

for $0 \le u \le 2\pi$, $0 \le v \le \pi$

II. $\vec{F}(\vec{r}(u,v)) = \langle a, b, c \rangle$

III. $\vec{r}_u = \langle -\sin u \sin v, \cos u \sin v, 0 \rangle$ $\vec{r}_v = \langle \cos u \cos v, \sin u \cos v, -\sin v \rangle$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin u \sin v & \cos u \sin v & 0 \\ \cos u \cos v & \sin u \cos v & -\sin v \end{vmatrix}$$

$\sin^2 u \sin v \cos v$

$= (-\sin v \cos u \sin v\, \vec{i} + 0\vec{j} + \sin u \sin v \sin u \cos v\, \vec{k})$

$- (\cos u \sin v \cos u \cos v\, \vec{k} + \sin u \sin v \sin v + 0\vec{i})$

$= -\sin^2 v \cos u\, \vec{i} + \sin u \sin^2 v\, \vec{j} + (\sin^2 u \sin v \cos v - \cos^2 u \cos v \sin v)\vec{k}$

$= \langle -\sin^2 v \cos u, \sin u \sin^2 v, -\sin v \cos v (\sin^2 u + \cos^2 u) \rangle$

$\sin^2 + \cos^2 = 1$

IV. $\iint_S \langle a,b,c \rangle \cdot \langle -\cos u \sin^2 v, -\sin u \sin^2 v, -\sin v \cos v \rangle \, dS$

$-\iint_S (a\cos u \sin^2 v + b \sin u \sin^2 v + c \sin v \cos v)\, dS$

$-\int_0^{2\pi} \int_0^{\pi} (a\cos u \sin^2 v + b \sin u \sin^2 v + c \sin v \cos v)\, dv\, du$

Use Maple to integrate...

$-\int_0^{2\pi} \frac{1}{2} b \sin u \pi + \frac{1}{2} a \cos u \pi = 0$

Great Job

2. Compute $\iint_S \langle x^2, 2y^2, 3z^2 \rangle \cdot d\mathbf{S}$ where S is the surface of the box with faces $x = 1$, $x = 2$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

Using Divergence Theorem

$$\int_0^1 \int_0^1 \int_1^2 (2x + 4y + 6z) \cdot dx\,dy\,dz$$

$$\int_0^1 \int_0^1 x^2 + 4xy + 6xz \Big|_1^2 \, dy\,dz$$

$$4 + 8y + 12z - (1 + 4y + 6z)$$

$$\int_0^1 \int_0^1 3 + 4y + 6z \, dy\,dz$$

$$\int_0^1 3y + 2y^2 + 6yz \Big|_0^1 \, dz = 3 + 2 + 6z$$

Nice Job

$$\int_0^1 5 + 6z \, dz$$

$$5z + 3z^2 \Big|_0^1 \quad 5 + 3 = 8$$