

Exam 2 Real Analysis 1 11/12/2004

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of the derivative of f at $x = a$.

2. State the Intermediate Value Theorem.

3. State and prove the Difference Rule for derivatives.

4. Give an example of a function which is differentiable and continuous on (a, b) but which does not satisfy the conclusion of the Mean Value Theorem.

5. Prove that if $f(x)$ is an even function defined on \mathbb{R} , then $f'(x)$ is an odd function.

6. State and prove the Squeeze Theorem for functions f , g , and h .

7. Prove that if a function f is differentiable at $x = a$, then f is continuous at $x = a$.

8. State and Prove Rolle's Theorem.

9. Does $\lim_{x \rightarrow \infty} \sin \sqrt{x}$ exist?

10. Suppose that f and g are differentiable functions defined on \mathbb{R} and that for some real numbers a and b (with $a < b$) we have $f(a) < g(a)$ and $f(b) > g(b)$. Does there have to exist a $c \in (a, b)$ for which $f(c) = g(c)$?