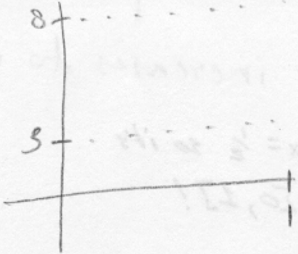
11 $f: [0,1] \to R$

a) bounded but $f \notin R[0,1]$



$$f(x) = \begin{cases} 3 & \text{when } x \text{ is rational} \\ 8 & \text{when } x \text{ is irrational.} \end{cases}$$

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since $m = 3$ and $M = 8$

which mean $L(P,f) \neq U(P,f)$

Great it not Reimann Integrable

2.) Do Problem 11b, §6.2.

Give an example of a function $f:[0,1]\to\mathbb{R}$ that is $f \in R[0,1]$ but not monotone.

Wonderful

$$f = (x-.5)^2$$

not increasing: $x_1 = 0$ $x_2 = .5$
$x_1 < x_2$, $f(x_1) = .25$ $f(x_2) = 0$

but not decreasing: $x_1 = .5$ $x_2 = 1$
$x_1 < x_2$, $f(x_1) = 0$ $f(x_2) = .25$

so not monotone.

But f is Riemann Integrable by Theorem 6.2.4 because it is continuous on $[0,1]$.

c) $f: [0,1] \to \mathbb{R}$ $\quad f(x) = \begin{cases} (x-0.5)^2 & \text{if } x \neq 0.5 \\ 0.25 & \text{if } x = 0.5 \end{cases}$

works