.9 #28 ① Prop: If $a, b \in \mathbb{Q}$ with $ab > 0$ & $a < b$, then $1/a > 1/b$.

Proof: Well, we know $a < b$. We can divide both sides by ab since $ab > 0$ Yes!, therefore we know $ab \neq 0$ & ab is not negative and division is allowed because we're not dividing by zero (no-no!) and we are for sure dividing by a positive so the inequality stays the same.

5

$$a < b$$

$$\frac{\cancel{a}}{ab} < \frac{\cancel{b}}{a\cancel{b}}$$

$$\frac{1}{b} < \frac{1}{a} \Rightarrow \frac{1}{a} > \frac{1}{b}$$

$\therefore$ the proposition is true!

Nice

#2: 1.9 #33.

If $a>1$, then $a^{n+1} > a^n > 1$ for all $n \in \mathbb{N}$.

Using Proof by Induction.

Basis: $n=1$: $a^{1+1} > a^1 > 1$

we know that $a>1$

$$a \cdot a > a \cdot 1 \quad \text{A12}$$

$$a^2 > a$$

$$\text{so } a^2 > a > 1$$

Assume: $a^{n+1} > a^n > 1$

Prove: $a^{(n+1)+1} > a^{n+1} > 1$

We know that $a^{n+1} > a^n > 1$ by our assumption.

$$a \cdot a^{n+1} > a \cdot a^n > a \cdot 1 \quad \text{A12}$$

$$a^{n+2} > a^{n+1} > a > 1$$

$$a > 1$$

$$a^{(n+1)+1} > a^{n+1} > 1. \ \square$$

Beautiful

Q.E.D.

$a, b \in \mathbb{R}$

$|ab| \leq |a| \cdot |b|$

If $a \geq 0$ and $b \geq 0$

$|ab| \overset{?}{\leq} |a| \cdot |b|$

$a \cdot b \overset{?}{\leq} |a| \cdot |b|$ by def

$a \cdot b \overset{?}{\leq} a \cdot b$ by def since a & b are pos.

$a \cdot b = a \cdot b$

If $a > 0$ $b < 0$

$|a \cdot b| \overset{?}{\leq} |a| \cdot |b|$ $ab < 0$ since $b < 0$ and $a > 0$ see #28

$a \cdot b \overset{?}{\leq} |a| \cdot |b|$ but $|ab| = -ab > 0$ by def

$-a \cdot b \overset{?}{\leq} a \cdot -b$ $|a| = a$ $|b| = -b > 0$ by def

$-a \cdot b \leq -a \cdot b$ comm.

$-a \cdot b = -a \cdot b$

If $a < 0$ $b > 0$

Same as above

If $a < 0$ $b < 0$

$|a \cdot b| \overset{?}{\leq} |a| \cdot |b|$

$a \cdot b \overset{?}{\leq} |a| \cdot |b|$ $a \cdot b > 0$ since $a, b < 0$ see #28

$|a \cdot b| = ab$ by def

$a \cdot b \overset{?}{\leq} -a \cdot -b$ $|a| = -a$, $|b| = -b$ by def

$a \cdot b \overset{?}{\leq} (-1)(-1) a \cdot b$

$a \cdot b = a \cdot b$

For all cases w/ $a, b \in \mathbb{R}$

$|a \cdot b| \leq |a| \cdot |b|$

but, more exactly $|a \cdot b| = |a| \cdot |b|$ $\square$

Yes!

#4

1.9.46 Proposition: If $a, b \in \mathbb{R}$, then $|a-b| \leq |a| + |b|$.

Proof: Well, $|a-b| = |a+(-b)| \leq |a| + |-b| = |a| + |b|$, with the inequality being a simple application of the triangle inequality to the real numbers a and $-b$. $\square$