Prove: Prove that the derivative of \( F(x) = x^n \) is \( F'(x) = nx^{n-1} \) for all \( n \in \mathbb{N} \).

Proof: Well, prove this by mathematical induction.
So, show that \( P(1) \) is true. Let \( g(x) = x^1 \). So by the definition of \( g'(x) \) we get:
\[
\lim_{x \to a} = \frac{g(x) - g(a)}{x - a} = \frac{x^1 - a^1}{x - a} = 1 \text{ which is the same as } 1x^0.
\]
So \( P(1) \) is true! Yay!! Now assume \( P(k) \) is true, which is if \( f(x) = x^k \) then \( f'(x) = kx^{k-1} \). We need to prove \( P(k+1) \) is true. So let \( h(x) = x^{k+1} \) or it can be written as \( x^k \cdot x^1 \). So using the product rule, which is \( F'(x)g(x) + f(x)g'(x) \), we get:
\[
h'(x) = (kx^{k-1}) \cdot x^1 + x^k \cdot 1 \text{ (we know the derivative of } x^k \text{ is } kx^{k-1} \text{ by inductive hypothesis, & we know } x^1 \text{ derivative is } 1 \text{ because we proved it earlier.) which can be rewritten as }
\]
\[kx^{k-1+1} + x^k \text{ or } kx^k + x^k \text{ which is the same as } (k+1)x^k \text{ by addition.}
\]
So \( h'(x) = (k+1)x^k \). So \( P(k+1) \) is true! Woahoo!!
Therefore if \( F(x) = x^n \) then we know \( F'(x) = nx^{n-1} \) is true for all \( n \in \mathbb{N} \). \( \square \)

Very nice job!
Well, the definition of the derivative is \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \).

We want to find \( f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} \)

\[ = \lim_{h \to 0} \frac{f(-(x-h)) - f(-(x))}{h} \]

Since \( f(x) \) is odd, then by the definition of an odd function, \( f(-x) = -f(x) \). Thus

\[ f'(-x) = \lim_{h \to 0} \frac{-f(x-h) - (-f(x))}{h} \]

Let \( p = -h \)

\[ = \lim_{-p \to 0} \frac{-f(x+p) + f(x)}{-p} \]

\[ = \lim_{-p \to 0} \frac{f(x+p) - f(x)}{p} \]

But since \(-p\) is approaching zero, it doesn't matter if \( p \) is negative or positive.

Therefore \( f'(-x) = \lim_{p \to 0} \frac{f(x+p) - f(x)}{p} \)

and by definition

\[ \lim_{p \to 0} \frac{f(x+p) - f(x)}{p} = f'(x) \]

So \( f'(-x) = f'(x) \), which by definition \( f \) is an even function.