Prove: Prove that the derivative of $F(x)=x^n$ is $F'(x)=nx^{n-1}$ for all $n \in \mathbb{N}$. Hallie

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Proof: Well, Prove this by mathematical induction. So, show that $P(1)$ is true. Let $g(x)=x^1$. So by the definition of $g'(x)$ we get $\lim_{x \to a} = \frac{g(x)-g(a)}{x-a} =$

$\frac{x^1-a^1}{x-a} = 1$ which is the same as $1x^0$.

So $P(1)$ is true! Yay!! Now assume $P(K)$ is true, which is if $f(x)=x^K$ then $f'(x)=Kx^{K-1}$. We need to prove $P(K+1)$ is true. So let $h(x)=x^{K+1}$ or it can be written as $x^K \cdot x^1$. So using the Product rule, which is $F'(x)g(x)+f(x)g'(x)$, we get $h'(x)=(Kx^{K-1}) \cdot x^1 + x^K(1)$ (we know the derivative of x^K is Kx^{K-1} by inductive hypothesis & we know x^1 derivative is 1 because we proved it earlier.) which can be rewritten as

$Kx^{K-1+1} + x^K$ or $Kx^K + x^K$ which is the same as $(K+1)x^K$ by addition.

So $h'(x)=(K+1)x^K$. So $P(K+1)$ is true! Woohoo!!

Therefore if $F(x)=x^n$ then we know $F'(x)=nx^{n-1}$ is true for all $n \in \mathbb{N}$. □

Very nice job!

2 Well, the definition of the derivative is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

We want to find $f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h}$

$$= \lim_{h \to 0} \frac{f(-(x-h)) - f(-(x))}{h}$$

Since $f(x)$ is odd, then by the definition of an odd function, $f(-x) = -f(x)$. Thus

$$f'(-x) = \lim_{h \to 0} \frac{-f(x-h) - -f(x)}{h}$$

let $p = -h$

$$= \lim_{-p \to 0} \frac{-f(x+p) + f(x)}{-p}$$

$$= \lim_{-p \to 0} \frac{f(x+p) - f(x)}{p}$$

But since $-p$ is approaching zero, it doesn't matter if p is negative or positive.

Therefore $f'(-x) = \lim_{p \to 0} \frac{f(x+p) - f(x)}{p}$

and by definition

Excellent

$$\lim_{p \to 0} \frac{f(x+p) - f(x)}{p} = f'(x)$$

So $f'(-x) = f'(x)$, which by definition f is an even function.