

**Problem Set 9****Real Analysis 1****Due 11/23/2004**

*“Whatever can go wrong...”*

Each problem is worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Can there exist a function  $f$ , interval  $[a, b]$ , and partition  $P$  such that  $L(P, f)$  is greater than the true value of the integral of  $f$  on  $(a, b)$ ?
2. Can there exist a function  $f$ , interval  $[a, b]$ , and partition  $P$  with representative points  $c_i$  such that  $S(P, f)$  is greater than  $U(P, f)$ ?
3. Can there exist a function  $f$ , interval  $[a, b]$ , and partitions  $P$  and  $Q$  with  $Q$  having more subintervals than  $P$ , but with  $L(P, f) < L(Q, f)$ ?
4. Let  $R(P, f)$  be the traditional right-hand sum for  $f$  with the partition  $P$ . Can there exist a function  $f$ , interval  $[a, b]$ , and partition  $P$  with refinement  $P'$  such that  $R(P, f)$  is closer to the true value of the integral of  $f$  on  $(a, b)$  than  $R(P', f)$ ?

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