Problem Set 2 Real Analysis 1 9/8/2004

1. No, I don't think it's a very good definition. First of all, notice that it's essentially different from the standard definition: $f(x) = x$ counts as a bounded function under this proposed definition (since for any $a \in \mathbb{R}$, $M = |a| + 1$ serves as a bound in this sense). But just because it's different from the normal definition of bounded doesn't make it bad – it might be better, or just different. But what <u>does</u> make it bad is that <u>all</u> functions count as bounded in this sense (since for any $a \in \mathbb{R}$, $M = |f(a)| + 1$ serves as a bound in this sense). So if it's a property that every single function has, it might not be a very interesting property.

2.) Use mathematical induction to prove that n^2+n is even for all $n \in \mathbb{R}$.

If this problem was stated correctly, then it is FALSE. Counterexample: $(1.45)^2 + (1.45) = 3.5525$, which is not an even number. A number k is even if it can be written $k = 2 \cdot m$, where $m \in \mathbb{Z}$.

$$3.5525 = 2(1.77625)$$
$$1.77625 \notin \mathbb{Z}.$$

Yes!

I think that this problem should state "... for all $n \in \mathbb{N}$." instead of "for all $n \in \mathbb{R}$". I will now prove it as if it were stated my way.

Basis: $n=1$: $1^2+1=2$, $2=2\cdot 1$, $1 \in \mathbb{Z}$ ✓

Assume: n^2+n is even, so $n^2+n = 2\cdot m$ for some $m \in \mathbb{Z}$.

Prove: $(n+1)^2 + (n+1) \stackrel{?}{=} 2\cdot k$ for some $k \in \mathbb{Z}$.

$$n^2+2n+1+n+1 \stackrel{?}{=} 2\cdot k$$
$$n^2+n+2n+2 \stackrel{?}{=} 2\cdot k$$

we know that $n^2+n = 2\cdot m$, so we insert that in.

$$2m+2n+2 \stackrel{?}{=} 2\cdot k$$
$$2(m+n+1) \stackrel{?}{=} 2\cdot k$$

where $(m+n+1) = k \in \mathbb{Z}$, which makes it even.

By proof of induction I have shown that n^2+n is even for all $n \in \mathbb{N}$.

Beautiful.

#3) Prop: $5 + 8 + 11 + \dots + (3n+2) = \frac{1}{2}(3n^2 + 7)$ for all $n \in \mathbb{N}$. → doesn't work for $\mathbb{R}$.

Yup.

i.e. $\sum_{i=1}^{n} (3i+2) = \frac{1}{2}(3n^2 + 7n)$

first: for $n=1$: $\sum_{i=1}^{1} 3i+2 \overset{?}{=} \frac{1}{2}(3(1)^2 + 7(1))$

$$3(1)+2 \overset{?}{=} \frac{1}{2}(3+7)$$

$$\underline{5 = 5} \checkmark$$

so for $n=1$, $\sum_{i=1}^{n} (3i+2) = \frac{1}{2}(3n^2 + 7n)$

second: assume it's true for $n = k$,

i.e. $\sum_{i=1}^{k} (3i+2) = \frac{1}{2}(3k^2 + 7k)$

is it true for $n = k+1$?

$$\sum_{i=1}^{k+1} (3i+2) \overset{?}{=} \frac{1}{2}\left(3(k+1)^2 + 7(k+1)\right)$$

$$\sum_{i=1}^{k} (3i+2) + (3(k+1)+2) \overset{?}{=} \frac{1}{2}\left(3(k^2+2k+1) + 7k + 7\right)$$

by the above assumption

$$\frac{1}{2}(3k^2 + 7k) + (3k+3+2) \overset{?}{=} \frac{1}{2}(3k^2 + 6k + 3 + 7k + 7)$$

$$\frac{3}{2}k^2 + \frac{7}{2}k + 3k + 5 \overset{?}{=} \frac{3}{2}k^2 + 3k + \frac{3}{2} + \frac{7}{2}k + \frac{7}{2}$$

$$\frac{3}{2}k^2 + \frac{13}{2}k + 5 = \frac{3}{2}k^2 + \frac{13}{2}k + 5 \checkmark$$

so if it's true for $n=k$, it's true for $n = k+1$

$\therefore$ We have proved by mathematical induction that

$\sum_{i=1}^{n} 3n+2 = \frac{1}{2}(3n^2 + 7n)$ for all $n \in \mathbb{N}$. $\square$

Nice Job!