5–6 ▮▮▮ Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways: (a) by using Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

5. $g(x) = \int_1^x t^2\,dt$

6. $g(x) = \int_0^x (1 + \sqrt{t})\,dt$

7–18 ▮▮▮ Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

7. $g(x) = \int_0^x \sqrt{1 + 2t}\,dt$

8. $g(x) = \int_1^x \ln t\,dt$

9. $g(y) = \int_2^y t^2 \sin t\,dt$

10. $g(u) = \int_3^u \dfrac{1}{x + x^2}\,dx$

11. $F(x) = \int_x^2 \cos(t^2)\,dt$

$$\left[\textit{Hint: } \int_x^2 \cos(t^2)\,dt = -\int_2^x \cos(t^2)\,dt\right]$$

12. $F(x) = \int_x^{10} \tan\theta\,d\theta$

13. $h(x) = \int_2^{1/x} \arctan t\,dt$

14. $h(x) = \int_0^{x^2} \sqrt{1 + r^3}\,dr$

15. $y = \int_3^{\sqrt{x}} \dfrac{\cos t}{t}\,dt$

16. $y = \int_1^{\cos x} (t + \sin t)\,dt$

17. $y = \int_{1-3x}^1 \dfrac{u^3}{1 + u^2}\,du$

18. $y = \int_{e^x}^0 \sin^3 t\,dt$

19–42 ▮▮▮ Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral, or explain why it does not exist.

19. $\int_{-1}^3 x^5\,dx$

20. $\int_{-2}^5 6\,dx$

21. $\int_2^8 (4x + 3)\,dx$

22. $\int_0^4 (1 + 3y - y^2)\,dy$

23. $\int_0^1 x^{4/5}\,dx$

24. $\int_1^8 \sqrt[3]{x}\,dx$

25. $\int_1^2 \dfrac{3}{t^4}\,dt$

26. $\int_{-2}^3 x^{-5}\,dx$

27. $\int_{-5}^5 \dfrac{2}{x^3}\,dx$

28. $\int_\pi^{2\pi} \cos\theta\,d\theta$

29. $\int_0^2 x(2 + x^5)\,dx$

30. $\int_1^4 \dfrac{1}{\sqrt{x}}\,dx$

31. $\int_0^{\pi/4} \sec^2 t\,dt$

32. $\int_0^1 (3 + x\sqrt{x})\,dx$

33. $\int_{\pi}^{2\pi} \csc^2\theta \, d\theta$

34. $\int_{0}^{\pi/6} \csc\theta \cot\theta \, d\theta$

35. $\int_{1}^{9} \frac{1}{2x} \, dx$

36. $\int_{0}^{1} 10^x \, dx$

37. $\int_{1/2}^{\sqrt{3}/2} \frac{6}{\sqrt{1-t^2}} \, dt$

38. $\int_{0}^{1} \frac{4}{t^2+1} \, dt$

39. $\int_{-1}^{1} e^{u+1} \, du$

40. $\int_{1}^{2} \frac{4+u^2}{u^3} \, du$

41. $\int_{0}^{2} f(x) \, dx$ where $f(x) = \begin{cases} x^4 & \text{if } 0 \leqslant x < 1 \\ x^5 & \text{if } 1 \leqslant x \leqslant 2 \end{cases}$

42. $\int_{-\pi}^{\pi} f(x) \, dx$ where $f(x) = \begin{cases} x & \text{if } -\pi \leqslant x \leqslant 0 \\ \sin x & \text{if } 0 < x \leqslant \pi \end{cases}$