Exam 3 Calc 3 12/2/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Give a parametrization and bounds for a counterclockwise circle centered at the origin in the *xy*-plane with radius 3.

2. Find the divergence of the vector field $\mathbf{F}(x,y,z) = \langle -z, y^2, x \rangle$.

3. Evaluate $\int_C \langle y\sin x, -\cos x\rangle \cdot d\mathbf{r}$ where C is a line segment beginning at (0,1) and ending at (2π,2).

4. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = 5x\mathbf{i} - 2y\mathbf{j} + 7z\mathbf{k}$ and S is the surface of the outward-oriented cube bounded by the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$.

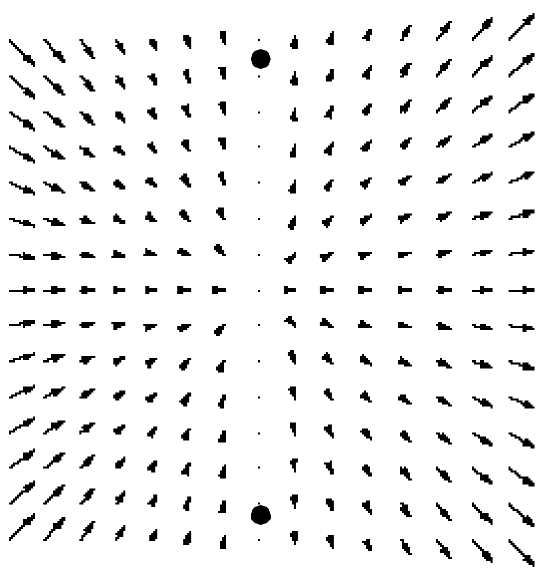
5. Let $\mathbf{F} = (z - 2y)\mathbf{i} + (3x - 4y)\mathbf{j} + (z + 3y)\mathbf{k}$, and let C be the circle $x^2 + y^2 = 4$, $z = 1$, oriented counterclockwise when viewed from above. **Use Stokes' Theorem** to find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

6. Show that curl(grad $f(x,y,z)$) = $\mathbf{0}$ for any function $f(x,y,z)$ with continuous second-order partials. Make it clear why it matters that the second-order partials be continuous.

7. Let $\mathbf{F}(x,y,z) = \langle -y,x \rangle$ and let C be a counterclockwise circle with radius a and center (h,k). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ in terms of the values of a, h, and k.

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, this stuff with pictures has me totally thrown. But so I guess this crappy book we've got is big on pictures or something, so they put this problem about from this picture if this vector field was conservative or not. So this girl Bunny who sits by me, she said it has to be conservative 'cause from where she put the dots, if you go from the one at the top to the one at the bottom, it doesn't matter which way you go. If you do it on a straight line, or half circle clockwise, or half circle counterclockwise, every way the integral comes out zero 'cause it's just across the arrows instead of with 'em or against 'em. I kinda remember the professor saying the stuff about how independent of path means conservative. So does that make sense?"

Explain clearly to Biff whether Bunny's reasoning is valid or not, and why.



9. Let $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and let S be the portion of the outward-oriented surface $x^2 + y^2 = 4$ between the planes $z = a$ and $z = a + h$. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

10. Let $\mathbf{F}(x,y) = \langle -y, x \rangle$. Let C be a line segment starting at (1,0) and ending at (a,b) for some constants a and b. Which values can a and b have in order for $\int_C \mathbf{F} \cdot d\mathbf{r}$ to work out to 3?

Extra Credit (5 points possible):

Compute $\int_C \left\langle \frac{x}{x^2+y^2}, \frac{-y}{x^2+y^2} \right\rangle \cdot d\mathbf{r}$ for C the counterclockwise circle centered at the origin of radius R.