Quiz 6 Calc 3 11/09/2005

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x,y) = y^3\,\mathbf{i} + 3xy^2\,\mathbf{j}$ where C is the top half of a circle centered at the origin beginning at (2,0) and ending at (-2,0).

$F(x,y): y^3 + 3xy^2$

$F_y(x,y) = 3x \tfrac{1}{3} y^3$

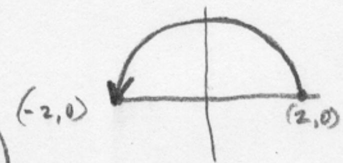
$= y^3 x$

$\frac{dv}{df}\, y^3 x = y^3$

So. ~~since~~

$y^3 x$ is a potential

we can use the fundamental Theorm



$f(b) - f(a)$

$f(-2,0) - f(2,0)$

$0 - 0 =$

$\boxed{0}$

Great

2. Let $\mathbf{F}(x,y) = -y\mathbf{i} + x\mathbf{j}$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for C the line segment beginning at (1,0) and ending at (2,3).

$\vec{F} = \langle -y, x \rangle$ (1,0) to (2,3)

partial wrt x = -y
patial (-y) wrt y = -1

partial wrt y = x
partial (x) wrt x = 1

Since mixed partials $(1 \neq -1)$ are not equal → need 5 step process

I. $x(t) = 1 + t$
$y(t) = 3t$
$0 \le t \le 1$ } $\vec{r} = \langle (1+t), 3t \rangle$

II $\vec{F}(\vec{r}(t)) = \langle -3t, (1+t) \rangle$

III. $\vec{r}\,'(t) = \langle 1, 3 \rangle$

IV $\int_0^1 \langle -3t, (1+t) \rangle \cdot \langle 1, 3 \rangle\, dt \rightarrow \int_0^1 (\cancel{-3t} + (3 + \cancel{3t}))\, dt$

$\int_0^1 3\,dt \rightarrow [3t]_0^1 = 3 - 0 = 3$

Excellent