

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x,y) = y^3 \mathbf{i} + 3xy^2 \mathbf{j}$ where C is the top half of a circle centered at the origin beginning at (2,0) and ending at (-2,0).

$$F(x,y) = y^3 + 3xy^2$$

$$F_y(x,y) = 3x \cdot \frac{1}{3} y^3$$

$$= y^3 x$$

$$\frac{dy}{dx} y^3 x = y^3$$

So since

$y^3 x$ is a potential
we can use the fundamental theorem

A hand-drawn diagram of a semicircle C in the upper half-plane, starting at (2,0) and ending at (-2,0). The points (2,0) and (-2,0) are labeled. An arrow on the curve points from (2,0) to (-2,0). To the right of the diagram, the calculation $f(b) - f(a)$ is written, with an arrow pointing to $f(-2,0) - f(2,0)$. Below this, the calculation $0 - 0 =$ is shown, with a box around the result. The word "Great" is written in a cursive font next to the diagram.

2. Let $\mathbf{F}(x,y) = -y\mathbf{i} + x\mathbf{j}$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for C the line segment beginning at (1,0) and ending at (2,3). $\vec{F} = \langle -y, x \rangle$

partial wrt $x = -y$
partial $(-y)$ wrt $y = -1$

partial wrt $y = x$
partial (x) wrt $x = 1$

Since mixed partials $(1 \neq -1)$ are not equal \rightarrow need 5 step process

I. $x(t) = 1+t$
 $y(t) = 3t$

$0 \leq t \leq 1 \} \vec{r} = \langle (1+t), 3t \rangle$

II $\vec{F}(\vec{r}(t)) = \langle -3t, (1+t) \rangle$

III. $\vec{r}'(t) = \langle 1, 3 \rangle$

IV $\int_0^1 \langle -3t, (1+t) \rangle \cdot \langle 1, 3 \rangle dt \rightarrow \int_0^1 (-3t + (3+3t)) dt$

$\int_0^1 3 dt \rightarrow [3t]_0^1 = 3 - 0 = 3$ ■

Excellent