Exam 1 Calc 2 9/15/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\int \left(\sin x - x^3 + x^{-1} + e^x \right) dx$.

2. **Set up an integral** for the average value of the function g(t) = x^2 on the interval [0,2].

3. Find the area of the region between $y = 6 - x^2$ and the x-axis.

4. Evaluate $\int \frac{1}{x(\ln x)^3}\,dx$.

5. A spring has a natural length of 16 inches, and 60 pounds hold it stretched to a length of 20 inches. How much work is done in stretching the spring from a length of 20 inches to a length of 22 inches?

6. Suppose that the region bounded by $y = \sin x$ and the x-axis between $x = 0$ and $x = \pi$ is rotated around the y-axis. **Set up** an integral for the volume of the resulting solid.

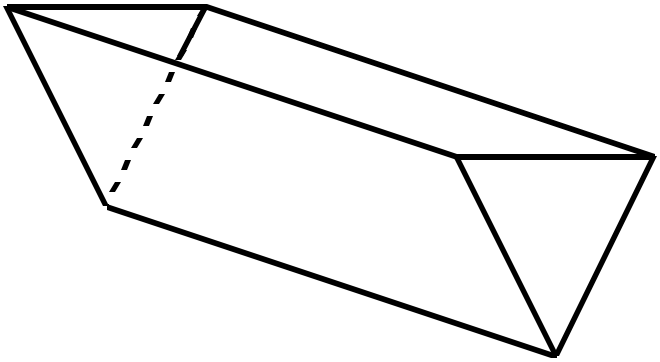
7. Suppose that the region bounded between $y = x^2$ and $y = x + 2$ is rotated around the x-axis. **Set up** an integral for the volume of the resulting solid.

8. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is the most totally confusing experience in my life. I studied the examples in the book a really long time and figured out how to do the shells thing and the washer thing, so I thought I was pretty good. But then our professor said something about problems where only one of them works, and I have no idea how you tell that, and we're not allowed to ask questions in lecture because there's like a thousand people. The only way I know how to tell which to use is by which section the homework problem is from!"

   Explain clearly to Bunny how you can tell when a problem needs to by done by shells rather than washers or discs.

9. Set up and evaluate an integral to show that the area of a trapezoid with height h and parallel sides with lengths b_1 and b_2 has area $\frac{h(b_1+b_2)}{2}$.

10. Suppose that a ditch shaped like a triangular prism (shown below) needs to be drained in order to free a baby giraffe whose leg is stuck in a culvert at one end of the ditch. The ditch is 40 feet long, 6 feet wide, and 6 feet deep, full of water weighing 62.5 pounds per ft^3. How much work is required to pump all the water up to the level of the top of the ditch?



Extra Credit (5 points possible):

A torus (the fancy name for the shape of a donut) is formed by revolving the region inside the circle $x^2 + y^2 = 1$ around the axis $x = 2$. What is the volume of this torus?