Exam 2 Calc 2 10/13/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Show how an appropriate trig substitution can transform the integral $\int \frac{1}{\sqrt{a^2 - x^2}} dx$ into the integral $\int 1 d\theta$.

2. Use midpoint approximations with $n = 3$ subdivisions to approximate $\int_1^4 \frac{1}{x} dx$.

3. **Set up** an integral for the circumference of a circle with radius 4.

4. Evaluate $\int \tan^3 x \sec x dx$.

5. Evaluate $\int \frac{1}{(x+4)(5-x)}\,dx$.

6. Let $p(x) = \begin{cases} 12x^2(1-x) & \text{if } 0 \leq \text{x} \leq 1 \\ 0 & \text{if x} \leq 0 \text{ or x} \geq 1 \end{cases}$. Find the mean of this probability density function.

7. Find the x coordinate of the center of mass of the first-quadrant portion of a circle centered at the origin with radius 3.

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "This calculus stuff is totally messed up. It's like, sometimes they do these integrals by they say it's symmetry and they just double part of it or whatever, right? But I did that on this one question on our test, where it was like the center of mass of a parabola, and got totally the wrong answer, and since it's multiple choice I don't even get partial credit. But I totally don't get why I was wrong, 'cause the right half and left half are exactly alike, right?"

   Explain clearly to Biff what's wrong with what he did, and under what circumstances using symmetry might be suspect.

9. Derive the formula $\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1-u^2} + C$ from the table of integrals.

10. Find the surface area resulting when the curve $y = e^{-x}$ to the right of $x = 0$ is rotated around the x-axis.

Extra Credit (5 points possible):

Derive the integral formula $\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$.