Exam 3   Calc 2   11/3/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. a) Convert the polar point $(5, \pi/2)$ to rectangular coordinates.

   b) Convert the rectangular point $(-2, -2)$ to polar coordinates.

2. Determine whether $y = x - x^{-1}$ is a solution to the differential equation $x\,y' + y = 2x$.

3. Find an equation for the line tangent to the curve with parametric equations

$$x = t^4 + 1$$
$$y = t^3 + t$$

at the point corresponding to $t = -1$.

4. Use Euler's method with a step size of $\Delta x = 0.2$ to approximate $y(0.4)$ for the differential equation $y' = 1 - xy$ if $y(0) = 0$.

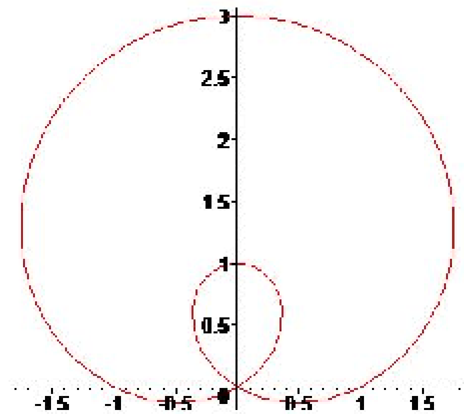
5. Find the vertices of the conic section $9x^2 + 25y^2 - 100y = 125$.

6. Bunny is a calculus student at Enormous State University, and she's still having some trouble. Bunny says "Ohmygod, this is so totally confusing! We learned about polar coordinates, and I thought I understood it, you know? But then our exam was all true/false, 'cause they have a machine do it, you know? So there was this question if any point in polar with the theta between zero and pi over two has to be in the first quadrant, you know? So, like, I said true, because that's where angles like that are, right? But it came back marked wrong, and the professor said not to come ask questions about the test because he'll lower your score. So what's up with *that*?

   Explain clearly to Bunny what the possibilities are for polar points with theta values between zero and pi over two.

7. Set up an integral for the area inside the inner loop of the curve with polar equation

$$r = 1 + 2\sin\,\theta.$$

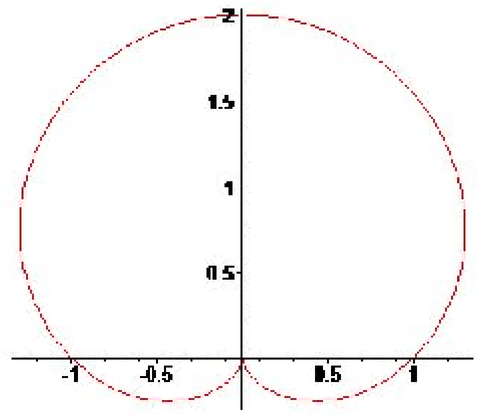


8. The concentration of some chemical in the bloodstream is given by $\frac{dC}{dt} = r - kC$. Find a general solution to this differential equation.

9. Use the parametric equations of an ellipse, $x = a\cos\theta$, $y = b\sin\theta$, $0 \leq \theta \leq 2\pi$, to find the area that it encloses.

10. Find the rectangular coordinates of the lowest point(s) on the graph with polar equation

$$r = 1 + \sin \theta.$$



Extra Credit (5 points possible): Find the area of one leaf of the rose $r = \cos n\theta$ where n is an integer greater than or equal to 2.