Exam 1 Calc 3 10/6/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to y.

2. Find an equation for the plane tangent to $z = x^2 + y^2$ at the point (3,–2).

3. Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

4. Suppose $z = f(x,y)$, where $x = g(s,t)$, $y = h(s,t)$, $g(1,2) = 3$, $g_s(1,2) = -1$, $g_t(1,2) = 4$, $h(1,2) = 6$, $h_s(1,2) = -5$, $h_t(1,2) = 10$, $f_x(3,6) = 7$, and $f_y(3,6) = 8$. Find $\partial z/\partial t$ when $s = 1$ and $t = 2$.

5. Let $f(x,y) = y^2/x$. Find the maximum rate of change of f at the point (2,4) and the direction in which it occurs.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is the most totally confusing experience in my life. The professor is such a total geek. It's like he keeps saying we're supposed to know what this stuff means instead of just finding the right answers, you know? But everybody knows math isn't like that. But just in case, I guess I should kinda have some clue, you know? So like directional derivatives are one of the things where he said we should know what it means, and I'm totally stumped. I mean, it's just a formula, right?"

   Explain clearly to Bunny what directional derivatives mean.

8. Find the maximum and minimum values of the function $f(x, y) = 3x - 2y$ subject to the constraint $x^2 + y^2 = 5$.

9. Show that the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ at the point (x_0, y_0, z_0) can be written as $\frac{x\,x_0}{a^2}+\frac{y\,y_0}{b^2}+\frac{z\,z_0}{c^2}=1.$

10. Determine the coordinates of the vertex of the paraboloid $z = x^2 + y^2 + axy + bx + cy + d$, where a, b, c, and d all represent constant real values. Is this vertex a maximum or minimum value, and how do you know?

Extra Credit (5 points possible):

Find values a and b with $a \leq b$ such that $\int_a^b \left(6 - x - x^2\right) dx$ is as large as possible.