

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find the gradient of the function  $f(x, y) = \ln(x^2 + y^4)$  at the point  $(2, 1)$ .

$$f(x, y) = \ln(x^2 + y^4)$$

$$f_x(x, y) = \frac{2x}{x^2 + y^4} \quad f_x(2, 1) = \frac{2(2)}{(2^2 + 1^4)} = \frac{4}{5}$$

$$f_y(x, y) = \frac{4y^3}{x^2 + y^4} \quad f_y(2, 1) = \frac{4(1^3)}{(2^2 + 1^4)} = \frac{4}{5}$$

$$\text{grad } f(2, 1) = \left\langle \frac{4}{5}, \frac{4}{5} \right\rangle$$

Excellent

2. Find the directional derivative of the function  $g(x, y) = x^2y - y^3$  in the direction of the vector  $\langle 3, -4 \rangle$  at the point  $(1, 2)$ .

$$\vec{u} = \frac{\langle 3, -4 \rangle}{\sqrt{3^2 + (-4)^2}} = \frac{\langle 3, -4 \rangle}{\sqrt{25}} = \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$$

$$g_x(x, y) = 2xy \quad g_x(1, 2) = 4$$

$$g_y(x, y) = x^2 - 3y^2 \quad g_y(1, 2) = -11$$

$$D = \frac{3}{5} \cdot 4 + \left(-\frac{4}{5}\right)(-11)$$

$$= \frac{12}{5} + \frac{44}{5}$$

$$= \frac{56}{5}$$

Great