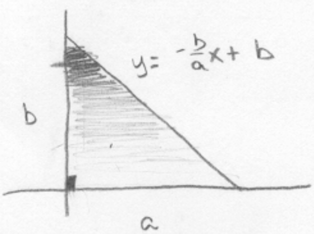
Quiz 5 **Calculus 3** **Due 10/24/2006**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find the center of mass of a right triangular region with legs of length a and b if the density is proportional to the distance from the leg of length a.

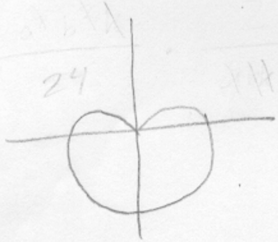


$$\bar{x} = \frac{\int_0^a \int_0^{-\frac{b}{a}x+b} kyx \, dy\,dx}{\int_0^a \int_0^{-\frac{b}{a}x+b} ky \, dy\,dx} = \frac{\frac{1}{24} a^2 \cancel{b^2} \cancel{k}}{\frac{1}{6} \cancel{a} \cancel{b^2} \cancel{k}} = \boxed{\frac{1}{4}a}$$

Nice!

$$\bar{y} = \frac{\int_0^a \int_0^{-\frac{b}{a}x+b} ky^2 \, dy\,dx}{\frac{1}{6} a b^2 k} = \frac{\frac{1}{12} \cancel{a} b^3 \cancel{k}}{\frac{1}{6} \cancel{a} \cancel{b^2} \cancel{k}} = \boxed{\frac{1}{2}b}$$

2. Find the center of mass of the cardioid $r = 4 - 4\sin\theta$ provided that the density is uniform.



$\rho(x,y) = k$

$0 \le r \le 4 - 4\sin\theta$

$0 \le \theta \le 2\pi$

Good

$$\bar{x} = \frac{\int_0^{2\pi} \int_0^{4-4\sin\theta} kx \, r\,dr\,d\theta}{\int_0^{2\pi} \int_0^{4-4\sin\theta} k \, r\,dr\,d\theta} = \frac{\int_0^{2\pi} \int_0^{4-4\sin\theta} k r^2 \cos\theta \, dr\,d\theta}{\int_0^{2\pi} \int_0^{4-4\sin\theta} kr \, dr\,d\theta} = \frac{0}{24k\pi} = 0$$

$\left(0, \frac{-10}{3}\right)$

$$\bar{y} = \frac{\int_0^{2\pi} \int_0^{4-4\sin\theta} ky \, r\,dr\,d\theta}{\int_0^{2\pi} \int_0^{4-4\sin\theta} k \, r\,dr\,d\theta} = \frac{\int_0^{2\pi} \int_0^{4-4\sin\theta} kr^2 \sin\theta \, dr\,d\theta}{\int_0^{2\pi} \int_0^{4-4\sin\theta} kr \, dr\,d\theta} = \frac{-80k\pi}{24k\pi} = -\frac{10}{3}$$