Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find the center of mass of a right triangular region with legs of length $a$ and $b$ if the density is proportional to the distance from the leg of length $a$.

\[ \bar{x} = \frac{\int_0^a \int_{-\frac{b}{a}x+b}^{\frac{b}{2}x+b} ky \, dy \, dx}{\int_0^a \int_{-\frac{b}{a}x+b}^{\frac{b}{2}x+b} ky \, dy \, dx} = \frac{\frac{1}{24} a^2 b^2 k}{\frac{1}{12} a b^3 k} = \frac{1}{4} a \]

\[ \bar{y} = \frac{\int_0^a \int_{-\frac{b}{a}x+b}^{\frac{b}{2}x+b} ky^2 \, dy \, dx}{\int_0^a \int_{-\frac{b}{a}x+b}^{\frac{b}{2}x+b} ky \, dy \, dx} = \frac{\frac{1}{12} a b^3 k}{\frac{1}{6} a b^2 k} = \frac{1}{2} b \]

2. Find the center of mass of the cardioid $r = 4 - 4\sin \theta$ provided that the density is uniform.

\[ p(x, y) = k \]
\[ 0 \leq r \leq 4 - 4\sin \theta \]
\[ 0 \leq \theta \leq 2\pi \]

\[ \bar{x} = \frac{\int_0^{2\pi} \int_0^{4-4\sin \theta} kr \, r \, d\theta \, dr}{\int_0^{2\pi} \int_0^{4-4\sin \theta} kr \, r \, d\theta \, dr} = \frac{0}{24k\pi} = 0 \]

\[ \bar{y} = \frac{\int_0^{2\pi} \int_0^{4-4\sin \theta} k y r \, r \, d\theta \, dr}{\int_0^{2\pi} \int_0^{4-4\sin \theta} k r \, r \, d\theta \, dr} = \frac{- \frac{80k\pi}{24k\pi}}{3} = \frac{-10}{3} \]

\[ (0, -\frac{10}{3}) \]