Quiz 7 **Calculus 3** **11/14/2006**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \langle 3x^2y + 2, x^3 - 2y \rangle$ and C is the arc of a unit circle centered at the origin traversing counterclockwise from (1, 0) to (0, -1).

Is there a potential function? $P_y = 3x^2 = 3x^2 = Q_x$ Yes!

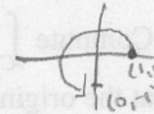
How about $f(x,y) = x^3y + 2x - y^2$.

So $\int_C \vec{F} \cdot d\vec{r} = \left[x^3y + 2x - y^2\right]_{(1,0)}^{(0,-1)}$

$= (0 + 0 - 1) - (0 + 2 - 0)$

$= -3$

2. Compute $\int_C \mathbf{G}\cdot d\mathbf{r}$ where $\mathbf{G}(x,y) = \langle 6xy, 2y^2\rangle$ and C is the arc of a unit circle centered at the origin traversing counterclockwise from (1, 0) to (0, -1).



I $\quad \vec{r}(t) = \langle \cos t, \sin t \rangle \quad 0 \le t \le \frac{3\pi}{2}$

II $\quad F(\vec{r}(t)) = \langle 6\cos t \sin t, 2\sin^2 t \rangle$

III $\quad \vec{r}\,'(t) = \langle -\sin t, \cos t \rangle$

IV $\quad \int_0^{\frac{3}{2}\pi} F(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$

V $\quad \int_0^{\frac{3}{2}\pi} \langle 6\cos t \sin t, 2\sin^2 t \rangle \cdot \langle -\sin t, \cos t \rangle\, dt$

$= \int_0^{\frac{3}{2}\pi} -6\cos t \sin^2 t, 2\sin^2 t \cos t)$

$= \int_0^{\frac{3}{2}\pi} -4\cos t (\sin t)^2\, dt$

$u = \sin t \qquad du = \frac{dt}{\cos t}$

$\frac{du}{dt} = \cos t$

$= \int_0^{\frac{3}{2}\pi} -4 \cancel{\cos t} (u^2) \frac{du}{\cancel{\cos t}}$

$= \int_0^{\frac{3}{2}\pi} -\frac{4}{3} u^3 \Big|_{t=0}^{t=\frac{3}{2}\pi}$

Nice!

$= -\frac{4}{3}(\sin t)^3 \Big|_0^{\frac{3}{2}\pi} = \left(-\frac{4}{3}\cdot -1\right) - (0) = \boxed{\frac{4}{3}}$