(Easier) Practice Quiz 8   Calc 3   11/14/2006

1. Compute the curl of the vector field $\mathbf{F}(x,y,z) = x^2\mathbf{i} - e^{xyz}\mathbf{j} + \cos y\ \mathbf{k}$.

I get $(-\sin y + xy\, e^{xyz})\ \mathbf{i} - yz\, e^{xyz}\ \mathbf{k}$.

2. Compute the divergence of the vector field $\mathbf{F}(x,y,z) = x^2\mathbf{i} - e^{xyz}\mathbf{j} + \cos y\ \mathbf{k}$.

I get $2x - xz\, e^{xyz}$.

(Harder) Practice Quiz 8 Calc 3 11/14/2006

1. Compute the curl of the vector field **F**(x,y,z) = $K\left(x^2+y^2+z^2\right)^{-3/2}\left(z\mathbf{i}+y\mathbf{j}+z\mathbf{k}\right)$.

It comes out to **0** (the vector 0, not the number 0). All the coefficients derivatives involved are 0, because the partial derivatives involved cancel each other. For instance, the coefficient of **i** includes $-3xy/(x^2+y^2+z^2)^{5/2}$ minus itself, producing 0. The **j** and **k** coefficients work out similarly.

This is actually a good thing -- if this sort of function is supposed to correspond to stuff coming out of the sun, we'd rather it come in straight lines without any "swirliness".

2. Compute the divergence of the vector field **F**(x,y,z) = $K\left(x^2+y^2+z^2\right)^{-3/2}\left(z\mathbf{i}+y\mathbf{j}+z\mathbf{k}\right)$.
[Note: Vector fields of this sort may be used to model photon flow from a star or neutrino flow from a supernova. Wow.]

Remember the quotient rule? Probably the least error-prone way to work this is by thinking of the coefficients as (for instance in the case of **i**): $\dfrac{Kx}{\left(x^2+y^2+z^2\right)^{3/2}}$. Then the derivative is

$$K\frac{1\left(x^2+y^2+z^2\right)^{3/2}-x\cdot\tfrac{3}{2}\cdot\left(x^2+y^2+z^2\right)^{1/2}(2x)}{\left(x^2+y^2+z^2\right)^3}$$

. This simplifies a lot – first divide top and bottom by $(x^2+y^2+z^2)^{1/2}$ and then collect like terms to get

$$K\frac{\left(x^2+y^2+z^2\right)-3x^2}{\left(x^2+y^2+z^2\right)^{5/2}}=K\frac{-2x^2+y^2+z^2}{\left(x^2+y^2+z^2\right)^{5/2}}$$

. Then it gets cool. That was just the x term.

When we add on the partials of the y and z terms (they look a lot like the x term, just with $-2y^2$ and $-2z^2$ respectively), everything cancels... at least as long as x, y, and z aren't all zero. So the divergence is 0 everywhere except at the origin, where its value is a function of K. As with #2, pretty reasonable considering the context.