Exam 1 Real Analysis 1 10/6/2006

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of convergence of a sequence.

2. State the definition of a decreasing sequence.

3. Give an example of a set with exactly two accumulation points.

4. Prove that for $f(x) = x^2$, $\lim_{x\to 5} f(x) = 25$.

5. State and prove the Squeeze Theorem for sequences.

6. State and prove the Monotone Convergence Theorem (proof of *either* case is acceptable).

7. Suppose that f and g are functions with both with domain $D \subseteq \mathbb{R}$. Prove that if $\lim_{x \to \infty} f(x) = A$ and $\lim_{x \to \infty} g(x) = B$ then $\lim_{x \to \infty} f \cdot g(x) = A \cdot B$.

8. We have repeatedly used the proposition that any finite set of real numbers has a smallest element (one that's less than or equal to all the others). Prove this proposition.

9. Prove that if f and g are defined on (a, ∞) for some $a \in \mathbb{R}$, $\lim_{x \to \infty} f(x) = L$, and $\lim_{x \to \infty} g(x) = +\infty$, then $\lim_{x \to \infty} (f \circ g)(x) = L$.

10. Prove that no point outside the interval [0,1] is an accumulation point of [0,1].