Problem Set 10 Real Analysis Due 12/1/2006

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Prove Lemma 6.1.3.

2. Prove that $U(\mathcal{Q}, f) \leq U(\mathcal{P}, f)$ in Lemma 6.1.5.

3. Do Problem #3 in §6.1.

4. Do Problem #4 in §6.1.

5. Evaluate $\int_0^2 x^3\, dx$ using upper and lower sums.

6. Prove Theorem 6.2.2 in the case where the function f is decreasing.

7. Do Problem 10(b) in §6.2.

8. Prove that if f is a bounded function on $[a, b]$ and f is discontinuous at exactly one point, then f is Riemann integrable on $[a, b]$.

9. Prove Theorem 6.3.1(b).

10. Give an example of functions $f, g{:}[0,1] \to \mathbb{R}$ such that neither f nor g is Riemann integrable on [0,1], but $f + g$ is.