Problem Set 2 **Real Analysis** **Due 9/8/2006**

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Let $a, b \in \mathbb{R}$. Show that $|a - b| < \varepsilon$ for all $\varepsilon > 0$ if and only if $a = b$.

2. Show that if $0 < a < 1$, then $0 < a^n \leq a$ for all $n \in \mathbb{N}$.

3. Let $a, b \in \mathbb{R}$. Show that $|a - b| \geq |a| - |b|$.

4. Suppose that $\{a_n\}$ and $\{b_n\}$ are unequal in only a finite number of terms, and $\{a_n\}$ converges. Show that $\{b_n\}$ converges as well, or provide a counterexample.

5. Suppose that $\{a_n\}$ and $\{b_n\}$ are equal in an infinite number of terms, and $\{a_n\}$ converges. Show that $\{b_n\}$ converges as well, or provide a counterexample.

6. Show that the sequence $\{a_n\}$ converges to 0 if and only if the sequence $\{|a_n|\}$ converges to 0.