Problem Set 8 Real Analysis Due 10/27/2006

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Show that if f is continuous at $c \in [a,b]$ then for any sequence $\{x_n\}$ converging to c, the sequence $\{f(x_n)\}$ converges to $f(c)$.

2. Prove or give a counterexample: If f and g are continuous on $[a,b]$, then $f + g$ is continuous on $[a,b]$.

3. Prove or give a counterexample: If f and g are uniformly continuous on $[a,b]$, then $f + g$ is uniformly continuous on $[a,b]$.

4. Prove or give a counterexample: If $f:[a,b] \to \mathbb{R}$ is bounded, then f satisfies the intermediate value property.

5. Prove or give a counterexample: If $f:[a,b] \to \mathbb{R}$ ~~is one-to-one and~~ satisfies the intermediate value property, then f is continuous on $[a,b]$.

6. Prove or give a counterexample: f is continuous at a if and only if $\lim_{x\to a} f(x) = f(a)$.

7. Prove or give a counterexample: If f,g are continuous, satisfy $f(a) \leq g(a)$, and $f(b) \leq g(b)$, then there exists a $c \in [a,b]$ such that $f(c) = g(c)$.

8. Prove or give a counterexample: Suppose that a function f is continuous on an interval I. If $a, b \in I$ such that $f(a)f(b) < 0$, then there exists $c \in (a,b)$ such that $f(c) = 0$.