Exam 1 Calc 1 9/21/2007

Each problem is worth 10 points. For full credit provide complete justification for your answers.

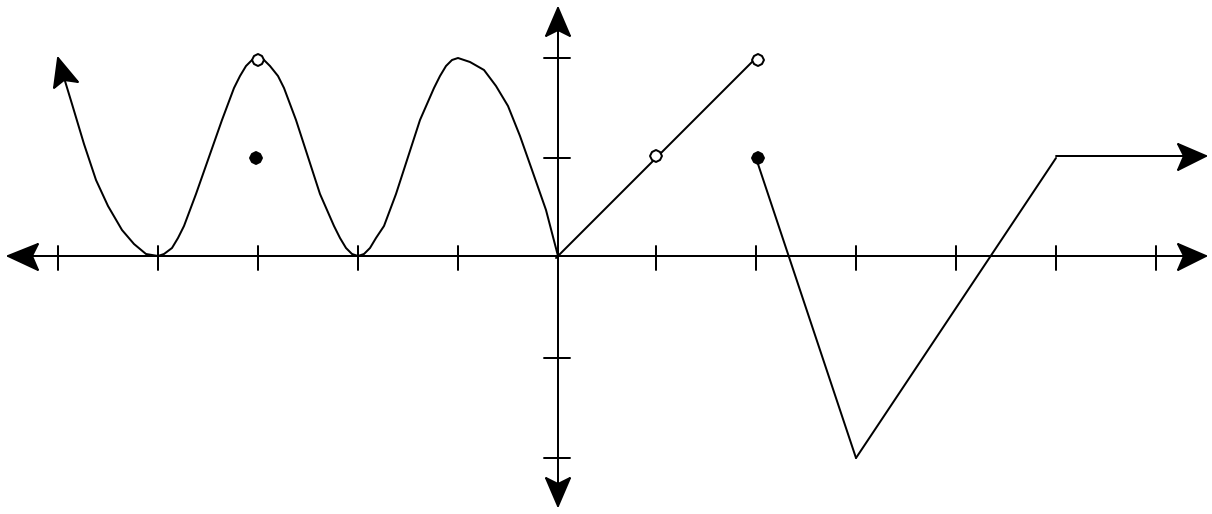
1. State the definition of the derivative of the function $f(x)$ at the point $x = a$.

Use the graph of $g(x)$ at the bottom of the page for problems 2 and 3:

2. Find the following limits:
   a) $\lim_{x \to -3^+} g(x)$
   b) $\lim_{x \to -3} g(x)$
   c) $\lim_{x \to 1} g(x)$
   d) $\lim_{x \to 2^-} g(x)$
   e) $\lim_{x \to 2} g(x)$

3. a) For which values of x does the function fail to be continuous?

   b) For which values of x does the function fail to be differentiable?



4. Evaluate the following limits:

   a) $\lim_{x \to 5^+} \frac{6}{5-x}$

   b) $\lim_{x \to \infty} \frac{6}{5-x}$.

5. Evaluate $\lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$.

6. If $f(x) = \dfrac{1}{2x-3}$, use the definition of the derivative to find $f'(x)$.

7. Evaluate $\lim_{x \to 0} \left(\frac{\sin(2+x) - \sin 2}{x} \right)$ to the nearest 0.001.

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap, I'm gonna have to drop Calc and my dad's gonna kill me for it. Our professor is off the deep end, though, and like half the class failed the first test. He said we deserved to fail, though, because of what this one guy in the class wrote for one question. Like, he wrote something about how you know the limit of a function is infinity when the height keeps getting bigger and bigger. The professor was saying all this stuff about how sometimes the heights get bigger and bigger without being infinity, but he pretty much lost me. I mean, it seemed to me like if every time you make x bigger, then the y gets bigger too, then that's when you say the limit is infinity, right? So anyway, I guess I'll drop out and flip burgers."

   Help Biff by explaining as clearly as you can what it means to say that the limit of a function (either as x approaches some fixed value a or as x approaches infinity) is infinity.

9. Evaluate $\lim_{x\to\infty}\left(x-\sqrt{x^2+ax+b}\right)$, where a and b are constants.

10. Suppose that $f(x)$ is a function whose derivative exists, and that $g(x) = f(x) + C$, where C is some constant. What can you say about $g'(x)$, and why?

Extra Credit (5 points possible):

Suppose that $f(x)$ is a function whose derivative exists, and that $g(x) = \frac{1}{f(x)}$. What can you say about $g'(x)$?