

Exam 3 Calc 1 11/9/2007

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\lim_{x \rightarrow \infty} \frac{e^x}{x^3}$

2. Given the following information about a continuous function $g(x)$, determine the intervals of increase and decrease and intervals of concavity of $g(x)$

	$(-\infty, -3)$	$(-3, 0)$	$(0, 5)$	$(5, +\infty)$
$g'(x)$	+	+	-	-
$g''(x)$	+	-	-	+

3. Let $f(x) = x^3 + 2x^2 - 1$. Find the absolute extrema of f on the interval $[-1, 1]$.

4. Suppose that a company's cost function is $C(x) = 40,000 + 300x + x^2$. Find the production level that will minimize the average cost.

5. A farmer wants to create a rectangular lot for his emu herd. He has 4800 feet of fence, and wants to have a dividing fence down the middle of the lot (parallel to one of the outside edges) to keep the males and females separated. What is the largest area that can be created?

6. Use Newton's Method with an initial approximation of $x_1 = 2$ to find x_2 , the second approximation to a solution of the equation $2 = x^3 - x^2$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. Our professor is gonna give us this test that's mostly true/false, 'cause he said he has lots of more important things to do than waste time grading our stuff, and so I thought that was cool at first 'cause I figured true/false would be easy, right? But then this girl in the class who sits by me, and I guess she studies a lot or something, she said she got a copy of a true/false Calc exam from last year or something, and I had no idea what was up with a lot of them. One was, like, if a function could have a max at a place where the derivative wasn't zero. I figured that one I knew for sure was false, 'cause if there's one thing I learned it's that you gotta take the derivative and set it equal to zero, right? But she said from the key it was true. How the heck could that be?"

Help Biff understand how a function could have a maximum value at a point where the derivative is non zero.

8. Find the exact x -coordinates of the global extrema of $f(x) = \frac{x-3}{x^2+1}$.

9. a) Let $f(x) = x^3 - 3x^2 + 2x$. Find all inflection points of f .

b) Suppose that f is a third degree polynomial which crosses the x -axis at three distinct point x_1 , x_2 , and x_3 . Show that f must have an inflection point at $(x_1 + x_2 + x_3)/3$.

10. Functions of the form $f(t) = ate^{-bt}$, for suitable values of constants a and b , can be used to model quantities such as the amount of caffeine in the bloodstream t minutes after drinking a caffeinated beverage. A study has determined that the maximum amount of caffeine in the bloodstream occurs 37.5 minutes after ingestion. Suppose that the peak amount of caffeine in a person's bloodstream is 50mg. What values for the constants a and b would produce a suitable function to model this situation?

Extra Credit (5 points possible):

What is $\lim_{x \rightarrow 0} (1+x)^{1/x}$?