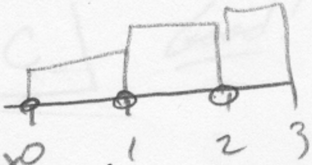
Exam 4 Calc 1 11/30/2007

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find L_3, the left-hand Riemann sum with 3 subdivisions, for $\int_0^3 \left(2x^2+3x+2\right) dx$.

$$\Delta x = \frac{3-0}{3} = 1$$



$$L_3 = f(0)\cdot\Delta x + f(1)\cdot\Delta x + f(2)\cdot\Delta x$$

$$= 2\cdot 1 + 7\cdot 1 + 16\cdot 1$$

$$\boxed{L_3 = 25}$$

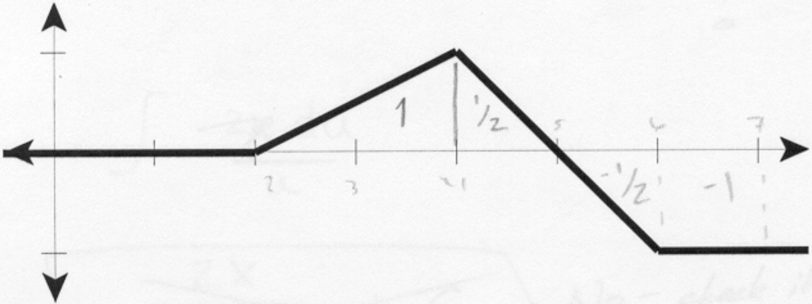
Great!

2. For the function $f(x)$ whose graph is shown below, find

a) $\int_2^5 f(x)\,dx$ $\boxed{1.5}$

b) $\int_2^7 f(x)\,dx$ $\boxed{0}$

Good



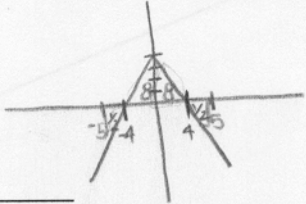
3. Evaluate $\int\left(e^x+2-6x^2+\frac{1}{\sqrt{x}}+\sin x\right)dx$.

$$= e^x + 2x - \frac{6x^3}{3} + \frac{2x^{1/2}}{1} - \cos x + C$$

$$\boxed{= e^x + 2x - 2x^3 + 2\sqrt{x} - \cos x + C}$$

Good

4. a) Evaluate $\int_{-5}^{5}\left(4-|x|\right)dx$ [Hint: Interpret it graphically].



$A\Delta = \frac{1}{2}bh$

$A = \frac{1}{2}(4)(4) = 8$

$\frac{1}{2}(1)(1)$

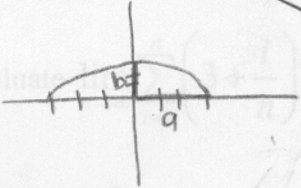
$-\frac{1}{2}+8+8-\frac{1}{2}$

$\boxed{15}$

b) Evaluate $\int_{-3}^{3}\sqrt{9-x^2}\,dx$ [Hint: Interpret it graphically].

equation of a circle

Excellent!



A of circle $= \pi r^2 = \pi(3)^2 = 9\pi$

• Divide by 2 ($\frac{1}{2}$ of a circle) $= \boxed{\frac{9\pi}{2}}$

5. Evaluate $\int \frac{5x}{x^2+1}\,dx$.

$$u = x^2+1$$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2x} = dx$$

$$\int \frac{5\cancel{x}}{u}\,\frac{du}{2\cancel{x}}$$

$$\frac{5}{2}\int \frac{1}{u}\,du$$

$$\frac{5}{2}\ln u + C$$

$$\frac{5}{2}\ln(x^2+1) + C$$

check $\frac{5}{\cancel{2}}\left(\frac{1}{x^2+1}\cdot \cancel{2}x\right)$

$$\frac{5}{x^2+1}$$

Excellent!

6. Write an expression in sigma notation for R_n for $\int_1^3 (9-x^2)\,dx$.

$\Delta x = \frac{2}{n}$

$x_i = 1 + \frac{2i}{n}$

$R_n = \sum_{i=1}^{n} f\left(1+\frac{2i}{n}\right)\cdot\frac{2}{n}$

Great!

$= \sum_{i=1}^{n} \left(9-\left(1+\frac{2i}{n}\right)^2\right)\cdot\frac{2}{n}$

7. Evaluate $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(3+\frac{i}{n}\right)\left(\frac{1}{n}\right)$.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(3+\frac{i}{n}\right)\left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}+\frac{i}{n^2}\right)$$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{3}{n} + \sum_{i=1}^{n} \frac{i}{n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\sum_{i=1}^{n} 3 + \frac{1}{n^2}\sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\cdot 3n + \frac{1}{n^2}\cdot\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} \left(3 + \frac{n^2+n}{2n^2}\right)$$

$$= \lim_{n\to\infty} \left(3 + \frac{1\cancel{n^2}}{2\cancel{n^2}} + \frac{n}{2n^2}\right)$$

$$= \lim_{n\to\infty} \left(3 + \frac{1}{2} + \frac{1}{2n}\xrightarrow{}0\right)$$

$$= 3\tfrac{1}{2}$$

Well done!

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. I got our quiz back today in our discussion section and the idiots gave me zero points on this one question, and then the T.A. was saying there'd be a question like that on the exam tomorrow. But the thing is, they didn't even mark what I did wrong, so if I can't fix that step I got no clue what to do. Here, look at this BS!"

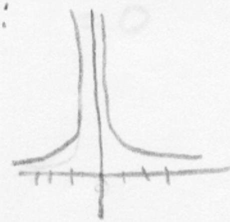
Biff's quiz is shown below. Help Biff understand what he did wrong, and how he might have answered differently.

2) Use the Fundamental Theorem of Calculus to evaluate $\int_{-2}^{3} 1/x^2\, dx$, if appropriate.

0/5

$$\int_{-2}^{3} \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_{-2}^{3} = \left(-\frac{1}{3}\right) - \left(\frac{1}{2}\right) = -\frac{5}{6}$$

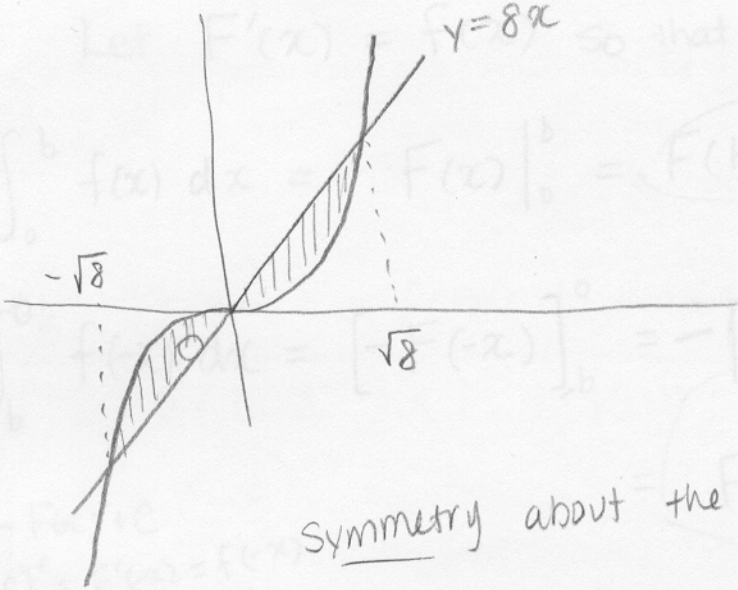
Well Biff you are not able to integrate this function from -2 to 3 because there is a vertical asymptote at $x=0$. The fundamental Therom of calculus states that the function must be continuous on the interval on wich you are trying to integrate. One way to see if the graph is continuous is to simply graph it. In doing this you get a graph like this:



Because of the vertical asymptote you can say that the $\int_{-2}^{3} \frac{1}{x^2}\, dx$ is simply undefined.

Excellent!

9. Find the area of the region bounded between the graphs of $y = 8x$ and $y = x^3$.



$8x = x^3$

$x^3 - 8x = 0$

$x(x^2 - 8) = 0$

$x = 0, \pm\sqrt{8}$

Symmetry about the y-axis.

Therefore, $\text{Area} = 2\int_0^{\sqrt{8}} (8x - x^3)\, dx$

$= 2\left[4x^2 - \frac{1}{4}x^4\right]_0^{\sqrt{8}}$

$= 2\left(4 \times 8 - \frac{1}{4} \times 8^2\right)$

$= 2(32 - 16)$

$= 32$

Nice

10. Show that for any continuous function $f(x)$, $\int_0^b f(x)\,dx = \int_{-b}^0 f(-x)\,dx$.

Look at $\int_{-b}^{0} f(-x)dx$, and let $u=-x$. Then $\frac{du}{dx} = -1$, so $dx = -du$, and when $x=-b$ we have $u=-(-b)=b$ and when $x=0$, $u=-(0)=0$. So

$$\int_{-b}^{0} f(-x)dx = \int_{b}^{0} f(u)\cdot -du$$

$$= -\int_{b}^{0} f(u)du$$

$$= \int_{0}^{b} f(u)du$$

But having the variable named u instead of x makes no difference to the value of the integral; $\int_0^b f(u)du = \int_0^b f(x)dx$.

So

$$\int_{-b}^{0} f(-x)dx = \int_0^b f(u)du = \int_0^b f(x)dx. \quad \square$$