Exam 3 Calc 2 11/2/2007

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Which of the following is (or are) a solution to the differential equation $\frac{dy}{dx} = 0.3y$?

   a) $y = 3e^{0.3t}$

   b) $y = 5e^{0.3t}$

   c) $y = e^{0.3t + 7}$

2. Set up an integral for the length of the cardioid with polar equation $r = 2 + \sin\theta$.

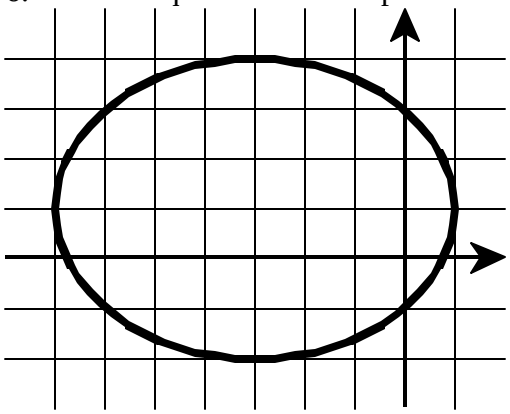
3. a) Convert the rectangular coordinates (–3, 3) to polar coordinates.

b) Convert the polar coordinates (6, $\pi/3$) to rectangular coordinates.

4. Identify the conic section $4y^2 - 9x^2 = 36$ as an ellipse, hyperbola, or parabola, and sketch a rough graph.

5. Let $\frac{dp}{dt} = 0.6p - 500$, and let $p(0) = 1000$. Use Euler's method with $\Delta t = 5$ to approximate $p(10)$ to the nearest hundred.

6. Find an equation for the ellipse shown.



7. Bunny is a calculus student at Enormous State University, and she has a question. Bunny says "You know, I've really been studying my calculus a lot more than before, and it's, like *amazing* how much more sense it makes when I actually study it! But so I was thinking about something the professor said in class about, like, with the parametric equations? She said that to find the place on a graph that's the farthest right you do this thing about where the slope is undefined, and I get that, and I've got the formula for the slope of those things memorized and everything. But I was thinking, like, couldn't you instead just do it like it was a Calc 1 problem, where you're looking for the maximum of the $x(t)$ function, 'cause the maximum x value would be the place farthest to the right, wouldn't it? So does that work?"

   Help Bunny by explaining how her suggestion does or does not match with the conventional approach mentioned by her professor.

8. Find the area inside both of the curves $r = \cos 2\theta$ and $r = \frac{1}{2}$.

9. The differential equation $\frac{dp}{dt} = kp - h$ can be used to model populations subject to steady harvesting, as for example with Iowa's deer population, where k and h are constants representing the relative rate at which a population is reproducing and the rate at which that population is being harvested. Find a general solution to this differential equation.

10. A curve called the folium of Descartes is defined by the parametric equations $x = \frac{3t}{1+t^3}$, $y = \frac{3t^2}{1+t^3}$. Find the exact area of the loop in this curve. [Stewart 5th, Ch. 10 PP #5]

Extra Credit (5 points possible): Show that the area between the folium of Descartes and its asymptote (the line $y = -x - 1$) is equal to the area inside the loop. [ibid.]